3.1 Systems of Equations

- Recall that $y = mx + b$ is an equation that we can interpret as a function.
- Now let's at it more generally as a <u>linear equation of 2 unknowns</u>

$$ax + by = c$$

*here 'b' does not refer to 'y-intercept'

- <u>def</u>: A <u>solution</u> of an equation is a pair of values that satisfy the equation. (When we 'plug in' these values the expression is true)

<u>ex</u> $5x + 3y = 2$

$(-2, 4)$ or $x = -2, y = 4$ is a solution because

$$5(-2) + 3(4) = 2$$
$$-10 + 12 = 2$$

✓

<u>non ex</u>

$(1, 2)$ is not a solution

$$5(1) + 3(2)$$
$$5 + 6 = 11 \neq 2$$

Solutions are not always unique

A single linear equation has infinitely many solutions

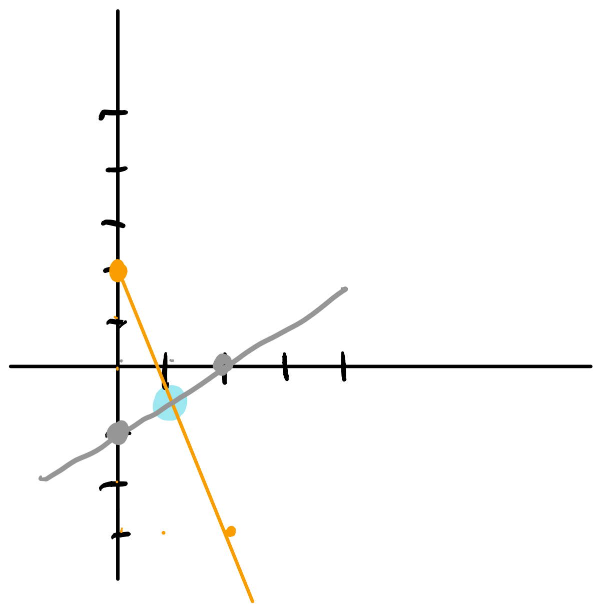
Because the graph of a function is the same set of solutions, then we already know how to plot solutions of a linear equation.

ex graph the solution set of $5x+2y=4$ and the solution set of $x-2y=2$.

- to make these equations familiar

$$5x+2y=4$$
$$2y=-5x+4$$
$$y=\frac{-5}{2}x+2$$

$$x-2y=2$$
$$-2y=-x+2$$
$$y=\frac{1}{2}x-1$$



$(1,-0.5)$ is a solution to both $5x+2y=4$ and $x-2y=2$

$(1,-0.5)$ is a solution to the system

$$5x+2y=4$$
$$x-2y=2$$

A system of two equations of two unknowns looks like

$$ax + by = c$$
$$dx + ey = f$$

A solution to a system is a solution to each equation in the system.

Although we found it graphically before we can also find it algebraically.

ex

$$5x + 2y = 4$$
$$x - 2y = 2$$

$$(5x + \cancel{2y}) + (x - \cancel{2y}) = 4 + 2$$
$$6x = 6$$
$$x = 1$$

$5x + 2y = 4$ and $x = 1$

$$5 \cdot (1) + 2y = 4$$
$$2y = -1$$
$$y = -\tfrac{1}{2}$$

$$\boxed{x = 1, \ y = -1/2}$$

ex

Find solution to:

$$3x+2y=1$$
$$6x-5y=5$$

$$-2(3x+2y)+(6x-5y) = -2(1)+5$$
$$\cancel{-6x}-4y+\cancel{6x}-5y = 3$$
$$-9y = 3$$
$$y = -1/3$$

- $3x+2y=1$ and $y=-1/3$

$$3x+2(-1/3)=1$$
$$3x+-2/3=1$$
$$3x=1+2/3=5/3$$
$$x=5/9$$

$$\boxed{\text{Solution is } (5/9, -1/3)}$$

Do two lines always intersect? <u>No</u>

Not all systems have a solution. If a system has no solutions we call it <u>inconsistent</u>

I claim that
$$\begin{aligned} 2x + y &= 4 \\ 4x + 2y &= -3 \end{aligned}$$
is inconsistent.

$2x+y=4 \longrightarrow y=-2x+4$

$4x+2y=-3 \longrightarrow y=-2x-3/2$

Same slope $\rightarrow$ parallel

$\rightarrow$ no solution

$-2 \cdot$ equation 1 + equation 2

$-2(2x+y) + 4x+2y = -2(4) + -3$

$-\cancel{4x} - \cancel{2y} + \cancel{4x} + \cancel{2y} = -8 - 3$

$0 = -11$

but $0 \neq -11$

$\rightarrow$ no solution

There might also be infinite solutions. We call these systems redundant.

$$-2\cdot(2x+y=4)$$
$$4x+2y=8$$
$$+$$
$$0x+0y=0$$
$$0=0$$
is true

is redundant, it has infinitely many solutions

We can still describe its infinite solutions as $y=-2x+4$ on $(x,-2x+4)$

ex In a past Brexit deal vote there were were 230 more votes against than for. There were 634 votes cast. How many voted for, how many against?

- assign variables: F be # for, A # against
- interpret data:

$$A-F=230$$
$$A+F=634$$

- Cancel out a variable

$$
\begin{array}{rl}
 & (A - F = 230) \\
+ & (A + F = 634) \\
\hline
 & 2A + \cancel{0F} = 864 \\
 & A = 432
\end{array}
$$

- Plug back in

$$A + F = 634$$

$$432 + F = 634$$

$$F = 634 - 432 = 202$$

- Interpret solution

432 voted against

202 voted for

ex One batch of cookies requires 3 cups flour and 1 cup of sugar. One batch of brownies require 1 cup of flour and 2 cups of sugar. You have 25 cups of flour and 20 cups of sugar and you want to use all your ingredients, how many batches of cookies and brownies should be made?

- Assign variables: C for # batches of cookies
  B # of brownies
- Interpret data:
  - we have 25 cups of flour

$$25 = 3C + 1B$$

  - We have 20 cups of sugar

$$20 = 1C + 2B$$

$$3C + 1B = 25$$
$$1C + 2B = 20$$

- Cancel out a variable

$$\begin{array}{r} 3C + 1B = 25 \\ -3(1C + 2B = 20) \\ \hline 0 + -5B = -35 \end{array}$$

$$B = 7$$

- Plug in

$$3C + 1B = 25 \longrightarrow 3C + 7 = 25$$
$$3C = 18 \longrightarrow C = 6$$

- Answer question:

  Make 6 batches of cookies

  7 batches of brownies

  to use all flour and sugar