Systems as Matrices

$$5x + 10y = 4$$
$$x - y = -3$$

$$\begin{bmatrix} 5 & 10 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ -3 \end{bmatrix}$$

↑ coefficient matrix

Augmented Matrix

$$\left[\begin{array}{cc|c} 5 & 10 & 4 \\ 1 & -1 & 3 \end{array}\right]$$

Looking at an augmented matrix we know that

- \# rows is # of equations
- \# cols − 1 is # of variables

$$\begin{bmatrix} 2 & 3 & -1 \\ 0 & 5 & 7 \end{bmatrix} \qquad \begin{aligned} 2x + 3y &= -1 \\ 0x + 5y &= 7 \end{aligned}$$

$$\begin{bmatrix} 1 & 13 & 1 & 0 \\ 2 & 0 & 9 & 1 \\ 0 & -1 & -2 & 10 \end{bmatrix} \qquad \begin{aligned} 1x + 13y + z &= 0 \\ 2x + 9z &= 1 \\ -1y - 2z &= 10 \end{aligned}$$

Some matrices are easier to 'read off' a solution

$$\begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 2 \end{bmatrix} \rightarrow \begin{aligned} x + 0y + 0z &= 4 \\ y &= 3 \\ z &= 2 \end{aligned}$$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \end{bmatrix} \rightarrow \begin{aligned} x &= 1 \\ y &= -1 \end{aligned}$$

$$\begin{bmatrix} 1 & 0 & 0 & a_1 \\ 0 & 1 & 0 & a_2 \\ 0 & 0 & 1 & a_3 \end{bmatrix}$$

Row Reduced Echelon Form

$$\begin{bmatrix} 1 & 0 & a_1 \\ 0 & 1 & a_2 \end{bmatrix} \qquad \left[\begin{array}{cccc|c} 1 & 0 & \cdots & 0 & a_1 \\ 0 & 1 & \ddots & \vdots & \vdots \\ \vdots & \ddots & \ddots & 0 & \vdots \\ 0 & \cdots & 0 & 1 & a_n \end{array}\right]$$

$$\begin{aligned} 3x + 5y &= 0 \\ -1x - 2y &= 4 \end{aligned} \longrightarrow \begin{bmatrix} 3 & 5 & 0 \\ -1 & -2 & 4 \end{bmatrix}$$

?

$$\begin{bmatrix} 1 & 0 & ? \\ 0 & 1 & ? \end{bmatrix} \longrightarrow \begin{aligned} x &= ? \\ y &= ? \end{aligned}$$

Gauss
Jordan
Elimination

In this class we will use technology to perform Gauss Jordan Elimination.

Will need to know

- how to create a corresponding matrix from a word problem
- how to use technology to get said matrix into row reduced echelon form
- how to interpret a RREF matrix to answer the original question.

A group of 33 people want to travel in cars that seat 4 people, cars that seat 5 people, and vans that fit 8 people. There are 6 people who will drive and 2 will drive vans. To fill every vehicle, how many 4-seat cars, 5-seat cars, and vans should get used?

Variables: U for 4 seat cars
I for 5 cars
V for van

equations:

$$4U + 5I + 8V = 33$$

$$U + I + V = 6$$

$$V = 2$$

augmented matrix:

$$\left[\begin{array}{ccc|c} 4 & 5 & 8 & 33 \\ 1 & 1 & 1 & 6 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

using technology

$$\begin{array}{c} \left[\begin{array}{cccc} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{array}\right] \\ \begin{array}{cccc} U & I & V & \end{array} \end{array}$$

interpret

$U = 3$ — 3 four seat cars

$I = 1$ — 1 five seat car

$V = 2$ — 2 vans

Redundant & Inconsistent Systems

Reducing an inconsistent system will result in a statement that is not true

$$\begin{array}{c} \begin{array}{cccc} x & y & z & \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 1 & 4 \end{bmatrix} \end{array} \rightsquigarrow \begin{array}{l} x = 2 \\ 0 = 3 \\ z = 4 \end{array}$$

← is not true

↑ inconsistent system

In a redundant system, reducing "gives a row of 0's"

$$\begin{array}{c} \begin{array}{cccc} x & y & z & \end{array} \\ \begin{bmatrix} 1 & 0 & 2 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \rightsquigarrow \begin{array}{l} x + 2z = 3 \\ y = 2 \\ 0 = 0 \end{array}$$

← is true, redundant

We can describe the infinite solutions of this system as

$$\left(x, 2, \frac{3-x}{2} \right)$$

$$x + 2z = 3$$
$$2z = 3 - x$$
$$z = \frac{3-x}{2}$$

$$\begin{array}{cccc} x & y & z & \\ \end{array}$$

$$\begin{pmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{pmatrix} \longrightarrow \begin{aligned} x + z &= 3 \\ y + z &= 2 \\ 0 &= 0 \end{aligned}$$

the system is redundant

$$(x, x-1, 3-x)$$

$$x + z = 3$$
$$z = 3 - x$$

$$y + z = 2$$
$$y = 2 - z$$
$$y = 2 - (3 - x)$$
$$= 2 - 3 + x$$
$$= x - 1$$