Decision Algorithm

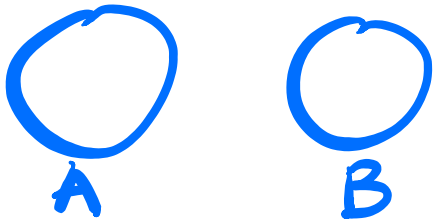
Additive & Multiplicative Principles

In general the formula for the size of the union of two sets is

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

If $n(A \cap B) = 0$ then

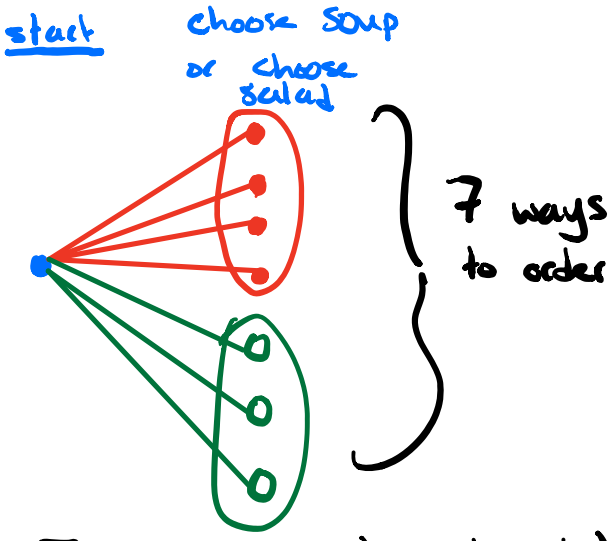
$$n(A \cup B) = n(A) + n(B)$$



If $n(A \cap B) = 0$ ie $A \cap B = \emptyset$ then we say A and B are <u>disjoint</u>.

The number of outcomes in many complicated experiments can be counted by breaking up the experiment into unions of disjoint sets and the cartesian product of sets.

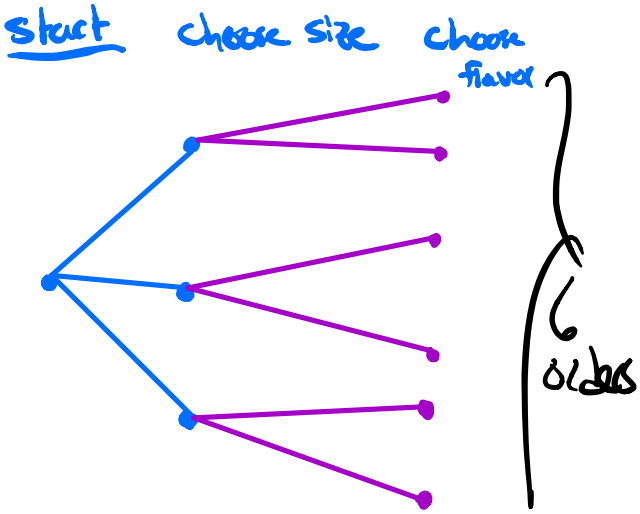
At a restaurant, part of your meal requires the choice of 1 soup from 4 options or 1 salad from 3 options.



$7 = n(\text{soups}) + n(\text{salads})$

additive principle

Ordering an ice cream cone requires choosing 1 of 3 sizes and then 1 of 2 flavors.



$6 = n(\text{sizes}) * n(\text{flavors})$

multiplicative principle

Decision Algorithm

The Decision Algorithm is a framework that classifies when to use the additive principle and when to use the multiplicative principle.

alternatives — describes cases that use the additive principle

steps — describes cases that use the multiplicative principle

ex

- Choosing a soup or salad — soup, salad are 2 alternatives
- Choosing size and flavor of ice cream — size, flavor are 2 steps

Step 1: 6 outcomes

- alternative 1: 3 outcomes
- alternative 2: 2 outcomes
- alternative 3: 1 outcome

} 3+2+1

Step 2: 4 outcomes

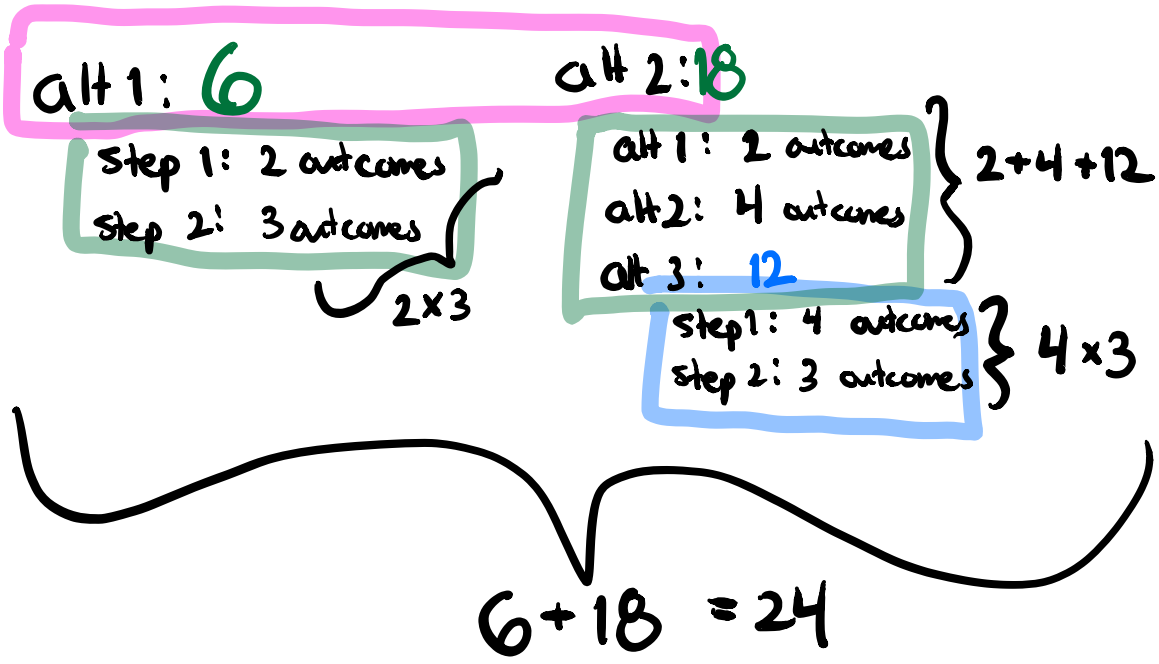
- alternative 1: 2 outcomes
- alternative 2: 2 outcomes

} 2+2

} 6 * 4 = 24

In this scenario there are 24 outcomes

To get a final # outcomes want to start at innermost layers and work outwards. Employ add. or mult. principle



This scenario described 24 outcomes

Using the Decision Algorithm Framework

At a build-your-own-burrito bar you need to choose white or brown rice, a kind of bean from 2 options (or no beans), choose a kind of main filling from 4 options, and choose a kind of salsa from 3 levels of heat.

Step 1: rice 2
- alt.1 white rice — 1 opt
- alt.2 brown rice — 1 opt

) 1+1

Step 2: beans 3
- alt.1 some kind — 2 opt.
- alt.2 no bean — 1 opt.

) 2+1

Step 3: main 4
- alt.1 some kind — 4 opts) 4

Step 4: salsa 3
- alt.1 some kind — 3 opt) 3

2 * 3 * 4 * 3

There are 72 ways to build a burrito

A pin consists of 4 digits (0-9) or a lowercase letter followed by 3 non-repeating digits (0-9).

ex 1 1 2 0

a 1 3 7

alt 1: 4 digit code 10000

step 1: first slot 10
s 2: second slot 10
s3: 3rd slot 10
s4: 4th slot 10

10·10·10·10

alt 2: letter then 3 n.r. digits 18,720

step: choose letter 26
step: choose # 10
step: choose # but not repeat 9
step: choose # avoid repeat 8

26*10*9*8*

10000 + 18720 =

There are 28,720 possible pins