Conditional Probability

A business conducts a survey to measure how its newest product is selling and how its ad is being viewed.

	saw ad	did not see ad	
bought item	100	200	300
did not buy item	300	1400	1700
	400	1600	2000

We've gone over how to answer questions like ...

$$P(\text{saw ad}) = 400/2000 = 0.2$$

$$P(\text{bought item}) = 1700/2000 = 0.85$$

$$P(\text{saw ad and bought item}) = 100/2000 = 0.05$$

But the business might have additional questions, like...

- Is it more likely someone would buy the item if they saw the ad?
- What's the probability that buyers saw the ad?

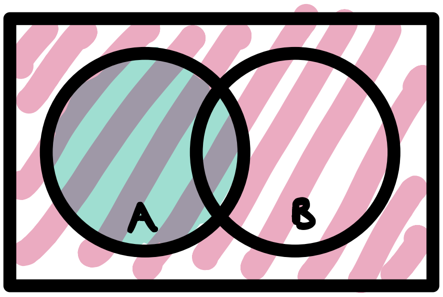
These questions highlight the concept of conditional probability, which measures the probability of an event given additional information.

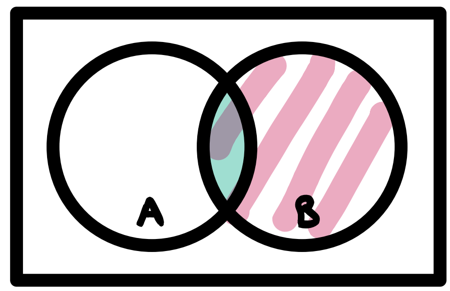
$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

'probability of A given B'

'assume B has happened, what's the probability that A will also happen?'

$$P(A) = \frac{n(A)}{n(S)} \qquad P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{n(A \cap B)}{n(B)}$$





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$$P(\text{bought} \mid \text{saw ad})$$

$$= \frac{n(\text{bought} \cap \text{saw})}{n(\text{saw})} = \frac{100}{400} = 0.25$$

$$P(\text{bought}) = 0.15$$

Ad increased likelihood of purchasing item.

Special Relations

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

- $P(A|B) = 0$ equivalent to $P(A \cap B) = 0$

  In this event we say A and B are mutually exclusive ie A and B never happen at the same time

- $P(A|B) = P(A)$ equivalent to $P(A \cap B) = P(A)P(B)$

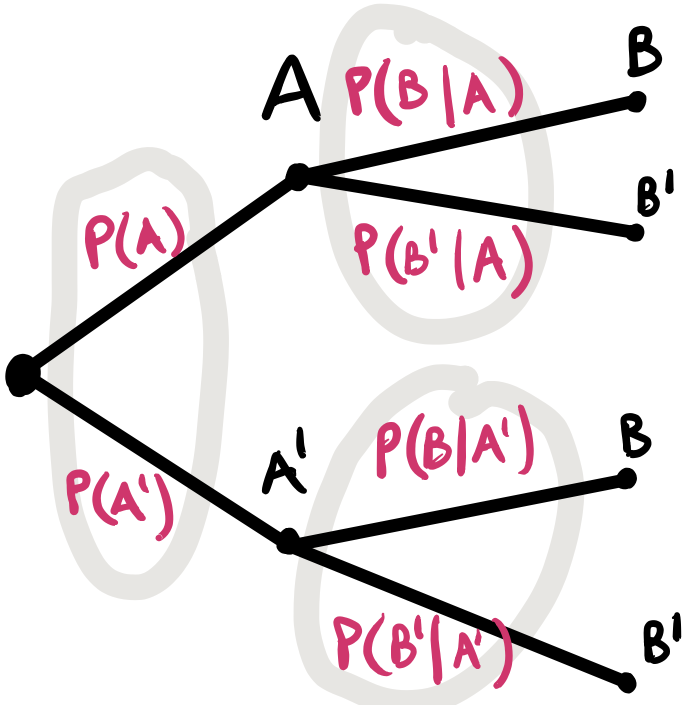
  In this event we say that A and B are independent ie one happening has no effect on the other happening

Mapping Probabilities

From $P(A|B) = \dfrac{P(A \cap B)}{P(B)}$ we get that

$$P(A \cap B) = P(B)P(A|B)$$

$$P(A \cap B) = P(A)P(B|A)$$



$P(A \cap B) = P(A)P(B|A)$

$P(A \cap B') = P(A)P(B'|A)$

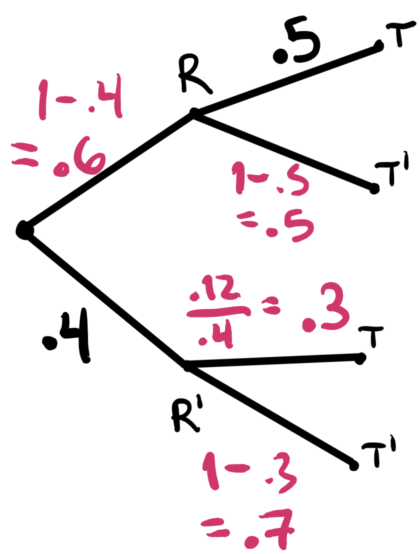
$P(A' \cap B) = P(A')P(B|A')$

$P(A' \cap B') = P(A')P(B'|A')$

$P(A) + P(A') = 1$,

$P(B|A) + P(B'|A) = 1$, $P(B|A') + P(B'|A') = 1$

$\bullet\ P(R') = .4 \qquad \bullet\ P(T|R) = .5 \qquad \bullet\ P(R' \cap T) = .12$



$P(R \cap T) = .6 \times .5 = .3$

$P(R \cap T') = .6 \times .5 = .3$

$P(R' \cap T) = .12$

$P(R' \cap T') = .4 \times .7 = .28$

To check: $P(R \cap T) + P(R \cap T') + P(R' \cap T) + P(R' \cap T') = 1?$

$.3 + .3 + .12 + .28 = 1$ ✓

2 Word Problems

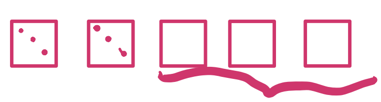
Yahtzee is played with 5 dice. A 'yahtzee' occurs when all 5 dice show the same number. You also have more than one roll and you may choose to not roll some of your dice.

(a) P('yahtzee' on first roll)?

(b) P('yahtzee' | 2 guaranteed 3s)?

(a) $$= \frac{n(\text{'yahtzee'})}{n(\text{roll 5 dice})} = \frac{6}{6\cdot6\cdot6\cdot6\cdot6} = \frac{1}{6^4}$$

(b) $$= \frac{n(\text{'yahtzee'} \cap \geq 2 \text{ 3s})}{n(\geq 2 \text{ 3s})} = \frac{1}{6\cdot6\cdot6} = \frac{1}{6^3}$$



A marble collector has 3 green, 4 red, and 5 blue marbles. You will blindly choose 5 marbles.

(a) $P(\text{get all green}) = \dfrac{C(3,3)C(9,2)}{C(12,5)} = \dfrac{1}{22} = 0.0\overline{45}$

(b) You cheat and peak, guaranteeing ≥ 1 green marble. How likely are you to get all green?

$$P(3 \text{ green} \mid \geq 1 \text{ green}) = \frac{n(3\text{green} \cap \geq 1 \text{ green})}{n(\geq 1 \text{ green})}$$

$$= \frac{C(3,3) \cdot C(9,2)}{736} = \frac{84}{736} = \frac{21}{184} \approx 0.114$$

$$n(\geq 1 \text{ green}) = n(5 \text{ marbles}) - n(0 \text{ green})$$

$$= C(12,5) - C(8,5) = 736$$