15 Apr 2019

- Tests will be back on wednesday
- M & W will continue probability 7.5 and 7.6
- Next monday class will not be held; a short reading assignment will be highly encouraged (more details to come)
- Wednesday of next week will include some overview/review for final
- Final is on Apr 29th; extra office hours offered. Either email me or take a chance and swing by

☆ to encourage studying and review, in the next few days all pasts webassigns will be opened and additional submissions granted; 25% of new point reduction

7.5

Suppose a survey found that

	saw ad	didn't see ad	
bought item	100	200	300
didn't buy	300	1400	1700
	400	1600	2000

We could ask questions like ...

what's propability of seeing ad and buying: $\frac{100}{2000} = 0.05$

$P(\text{saw ad}) = \frac{400}{2000} = 0.20$

$P(B) = \frac{300}{2000} = 0.15$

Other questions might be more informative:

- What's the probability someone who bought the ~~ad~~ item saw the ad?
- Is it more likely for someone to buy the item if they saw the ad?

Let's try to answer these:

- Prob of seeing ad if you're told that they bought it

	saw ad	didn't	
bought it	100	200	300

$$\frac{\text{people who saw and bought it}}{\text{total people who bought}} = \frac{100}{300} \approx 0.\overline{33}$$

- Prob of buying, if you're told they saw the ad

	saw ad
bought it	100
didn't buy	300
	400

$$\frac{\text{total that saw add and bought}}{\text{total that saw ad}} = \frac{100}{400} = 0.25$$

* note: the order is very important

- Prob of buying, if you're told they didn't see ad

	didn't see ad
B	200
B'	1400
	1600

so $\frac{200}{1600} = 0.125$

We can see that knowing more about the situation changes probabilities.

P(~~BOX~~ bought item) $\neq$ P(bought it, if you knew they saw the ad)

The form we're dealing with here is called conditional probability; it's asking about the probability of an event given some extra conditions

our concise notation is:

read as 'given'

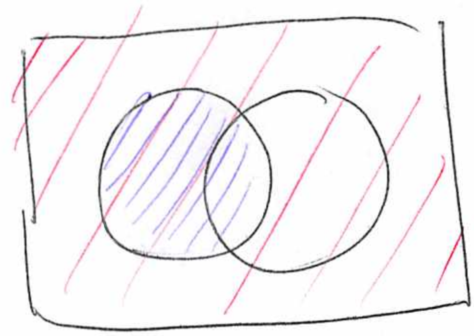
P(bought it if you're told they saw the add) = P(bought it | saw add) = P(B|A)

P(that a person saw the add if you're told they bought the item) = P(saw add | bought item) = P(A|B)

In general:

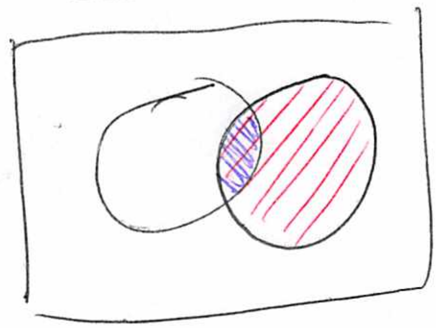
$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Why? well $P(A)$ is $\dfrac{\boxed{n(A)}}{\boxed{n(S)}}$



$$P(A|B) = \frac{\boxed{n(A \cap B)}}{\boxed{n(B)}}$$

only looks at outcomes were B happened



again! order matters

numerators equal

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \neq \frac{P(B \cap A)}{P(A)} = P(B|A)$$

these not always equal

What if we found $P(A|B) = 0$?

if $P(A|B) = \frac{P(A \cap B)}{P(B)}$ and is 0

then $P(A \cap B) = 0$.

This means that A and B never happen at the same time. So $P(A|B) = 0$ makes sense, because if we're told B has happened and that B and A never happen at same time, then A won't happen.

We call such events <u>mutually exclusive</u>

$P(A|A') = 0$ always

What happens if we know $P(A|B)$ and $P(B)$ but not $P(A \cap B)$?

if $$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

then $$P(A \cap B) = P(B) \cdot P(A|B)$$

probability of A and B happening is probability of B times probability of A happening if B has happened

<u>ex</u>

Suppose you knew there's a 50% chance of rain. Suppose you knew that 20% of rain storms have lightning. → 20% chance of lightning if you're told it's raining out.

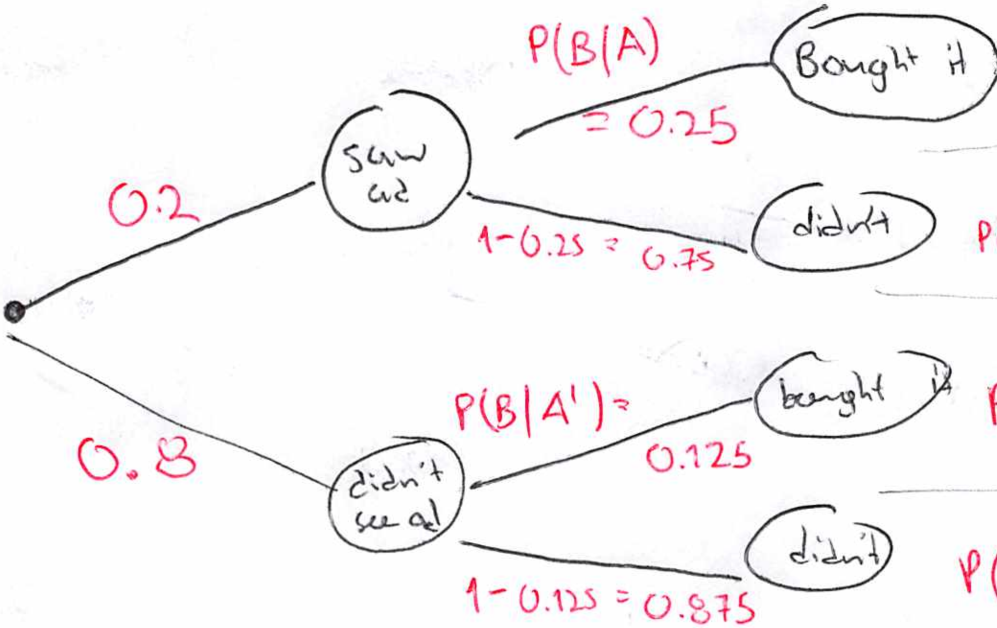
$$P(\text{lighting} \cap \text{rain}) = P(\text{rain})\, P(\text{lighting} | \text{rain})$$

$$= 0.5 * 0.2 = 0.1$$

One way to visualize this multiplicative property is through probability trees.

recall 1st example:

$P(\text{ad}) = 0.2$; $P(\text{Buying} \mid \text{ad}) = 0.25$; $P(\text{Buying} \mid \text{didn't seen ad}) = 0.125$



$P(A \cap B) = 0.2 * 0.25 = 0.05$

$P(A \cap B') = 0.2 * 0.75 = 0.15$

$P(A' \cap B) = 0.8 * 0.125 = 0.1$

$P(A' \cap B') = 0.8 * 0.875 = 0.7$

our tree starts with a single pt that branches into saw ad and didn't see ad

label probability of each branch

each node branches into bought or didn't buy; probabilities are our conditional ones

* we can multiply the probabilities of the branches to find prob of end result *

Does $P(A \cap B)$ ever equal $P(A) \cdot P(B)$?

well if $P(A \cap B) = P(B)\,P(A|B)$

$$\begin{array}{c} P(A \cap B) \\ \| \\ P(B \cap A) = P(A)\,P(B|A) \end{array}$$

so we would need

$P(A|B) = P(A)$ and
$P(B|A) = P(B)$

ie knowing that B occurred doesn't change the likelihood of A occurring and vice versa

this would mean the two events don't affect the other's chance of occurring

We call these events independent.

<u>def</u> if

$$P(A \cap B) = P(A)\,P(B)$$

(equivalently, if $P(A) = P(A|B)$ and $P(B) = P(B|A)$)

then A and B are independent events,

if they are not independent, they are dependent.

With our advertising example...

$P(\text{Ad} \cap \text{bought item}) = 0.05$

$P(\text{Ad}) \cdot P(\text{Bought item}) = 0.2 * 0.15 = 0.03$

So here seeing the ad and buying the item are dependent events.

How do we interpret dependence?

- $P(\text{bought item} \mid \text{ad}) > P(\text{bought item})$
  tells us someone is more likely to buy item if they saw the ad (good ads do this)
- $P(\text{bought item} \mid \text{ad}) < P(\text{bought item})$
  would mean that people who saw ad were less likely to purchase item. (bad ad)

What's an example of independent events?

flip a coin and roll a die.

H: flip a head; $P(H) = \frac{1 \cdot 6}{2 \cdot 6} = \frac{6}{12} = 0.5$ $\frac{1}{2}$

S: rolling a 6; $P(S) = \frac{2 \cdot 1}{2 \cdot 6} = \frac{2}{12} = 0.1\overline{66}$ $\frac{1}{6}$

$H \cap S$: flipping heads and rolling 6; $P(H \cap S) = \frac{1 \cdot 1}{2 \cdot 6} = \frac{1}{12} = \frac{1}{2} \cdot \frac{1}{6}$

So flipping a head and rolling a 6 are independent events

$P(S \mid H) = P(S)$ and similar for $P(H \mid S)$

- approximately 1 in 10 peeple are left handed
- there are 44 different US presidents and 8 were left handed
- Is left handedness and becoming president independent?

$$P(L) = \frac{1}{10} = 0.10$$

$$P(L \mid Pres) = \frac{8}{44} = 0.\overline{18}$$

$$P(L) \neq P(L \mid Pres)$$

these are not independent events