Refresher:

- How many ways to arrange 30 students into a line?

¿ are sets enough? NO $\Rightarrow$ Permutations

$$P(30,30) = 30!$$

- How many ways to choose 10 students from 15?

¿ are sets enough? yes $\Rightarrow$ combinations

$$C(15,10) = \frac{P(15,10)}{10!} = \frac{15!}{(15-10)!\,10!} = \frac{15!}{10!\,5!}$$

- Number of 3 digit long credit card security numbers?

¿ are sets enough? ... look at example security code

418 or 992 in general $\underbrace{\_\_}_{10 \text{ options}}\ \underbrace{\_\_}_{10 \text{ options}}\ \underbrace{\_\_}_{10 \text{ options}}$

so $10 \cdot 10 \cdot 10 = 10^3$

- Number of 4 digit bank pins starting with 7?

look at example: 7112, 7667, 7123

in general : $\underbrace{7}_{1 \text{ option}}\ \underbrace{\_\_}_{10 \text{ option}}\ \underbrace{\_\_}_{10 \text{ option}}\ \underbrace{\_\_}_{10 \text{ options}}$

$$1 \cdot 10 \cdot 10 \cdot 10 = 10^3$$

1 April

7.2

Suppose at a grocery store you examined apples, checking if they were bruised.

The <u>frequency</u> of bruised apples would be the number of bruised apples observed.

Comparing frequencies can be misleading...

at store 1: frequency of bruised apples was 20
at store 2: " " was 4

but at store 1 you checked 100 apples and at store 2 you only checked 5.

<u>Relative frequency</u> addresses this, it is

$\frac{\text{frequency}}{\text{size of entire sample}}$ $\quad P(E) = \frac{fr(E)}{N}$

$P(E)$ is relative frequency of event E and N is number trials.

So relative frequency of bruised apples...
at store 1 : $20/100 = 0.2$
at store 2 : $4/5 = 0.8$

We can interpret relative frequency as <u>estimated probability</u>.

The estimated probability of getting a bruised apple is much greater at store 2.

A survey of 50 students' birth month yields

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
f:	5	2	0	6	6	5	7	3	4	6	3	3
RF:	.1	.04	0	.12	.12	.1	.14	.06	.08	.12	.06	.06

Are estimated probabilities always good?

No, people born in March exist

The field of statistics studies ways to measure 'trustworthiness' of estimated probabilities.

Suppose you rolled a die 20 times and observed

	1	2	3	4	5	6
freq.	2	3	4	3	6	2
rel freq	.1	.15	.2	.15	.3	.1

$$fr(1) + fr(2) + fr(3) + \cdots + fr(6) = 20$$

$$P(1) + P(2) + \cdots + P(6) = 1$$

for a sample $S = \{s_1, \ldots, s_n\}$

1. $0 \leq P(s_i) \leq 1$
2. $P(s_1) + \cdots + P(s_n) = 1$
3. If E is a set of outcomes, eg $E = \{s_1, s_2, s_5\}$

$$P(E) = P(s_1) + P(s_2) + P(s_5)$$

(3 $\Rightarrow$ $P(S) = P(s_1) + P(s_2) + \cdots + P(s_n) = 1$)

What's the relative frequency of rolling an even #?

$$E = \{2, 4, 6\}$$

$$P(E) = P(2) + P(4) + P(6)$$

$$.15 + .15 + .1 = 0.4$$

As we increase our sample size, relative frequencies tend to stabilize.

flipping 2 coins : H $\approx$ 0.5

T $\approx$ 0.5

rolling a die : 1 $\approx$ $.1\overline{66}$ 1/6

2 $\approx$ $.1\overline{66}$

$\vdots$

6 $\approx$ $.1\overline{66}$ 1/6

Sum of 2 dice : 2 $\approx$ 0.027 ...

7 $\approx$ 0.1667

10 $\approx$ 0.08 ...

Where these numbers come from and how to calculate them without doing large amounts of trials is 7.3.

7.3

Using relative frequency as an estimated probability is just one <u>probability distribution</u> (ways to assign $P(S_i)$)

A <u>probability model</u> is a particular probability distribution that assigns

$P(S_i)$ as the predicted relative frequency done over a very large sample size.

If relative frequency gave estimated probability then modeled probability gives theoretical probability.

If each outcome is assumed to be equally likely, we can directly calculate the probability model.

coin flip : $S = \{H, T\}$ 2 outcomes, assign probability to each 1/2

$P(H) = 1/2$
$P(T) = 1/2$

die roll: $S = \{1, 2, 3, 4, 5, 6\}$ 6 outcomes

each outcome has probability $1/6$

$P(1) = 1/6 = .1\overline{66}$

$P(4) = 1/6$

$$P(\text{Even}) = P(\{2, 4, 6\})$$
$$= P(2) + P(4) + P(6)$$
$$= 1/6 + 1/6 + 1/6 = 3/6 = 1/2$$

Sum of 2 dice: $S = \{$

(1,1)	(1,2)	13	14	15	16
(2 1)	(2 2)	23	24	25	26
3 1	3 2	3 3	3 4	35	36
4 1	(4 2)	(4 3)	4 4	45	46
5 1	(5 2)	5 3	(5 4)	(5 5)	5 6
6 1	6 2	6 3	6 4	(65)	66)

$\}$

so each outcome has probability $\frac{1}{36}$

$P(\text{Sum is } 2)$

$P(\{(1,1)\}) = 1/36 \approx 0.02\overline{7}$

$P(\text{Sum is } 7) = P(\{(6,1), (5,2), (4,3), (3,4), (2,5), (1,6)\}) = 1/6 = 0.1\overline{6}$

$P(\text{Sum is } 10) = P(((6,4), (5,5), (4,6))) = \frac{3}{36} = \frac{1}{12} \approx 0.08\overline{3}$

In an experiment in which outcomes are equally likely, we model the probability of an event as

$$P(E) = \frac{\#\text{ of favorable outcomes}}{\#\text{ total outcomes}} = \frac{n(E)}{n(S)}$$

100 students entered a raffle with 1 random winner and there were

20 freshmen
30 sophomores
40 juniors
10 seniors

What's the probability of a freshman winning?

$$P(F) = \frac{n(F)}{n(S)} = \frac{20}{100} = 0.2$$

$$P(F \cup So) = \frac{n(F \cup So)}{n(S)} = \frac{n(F) + n(So) - n(F \cap So)}{n(S)}$$

$$= (20+30)/100 = 0.5$$

$$\frac{n(F)+n(S_o)-n(F\cap S_o)}{n(S)} = \frac{n(F)}{n(S)} + \frac{n(S_o)}{n(S)} - \frac{n(F\cap S_o)}{n(S)}$$

$$= P(F)+P(S_o)-P(F\cap S_o)$$

in general

$$P(A\cup B) = P(A)+P(B)-P(A\cap B)$$

* just like cardinality

Probability of not a senior winning?

$$P(S_e') = \frac{n(S_e')}{n(S)} = \frac{n(S)-n(S_e)}{n(S)} = \frac{100-10}{100} = 0.9$$

$$\rightarrow \frac{n(S)}{n(S)} - \frac{n(S_e)}{n(S)} = P(S)-P(S_e)$$

$$= 1 - P(S_e)$$

in general

$$P(E') = 1-P(E)$$

A survey of graduating high school seniors found

68% were going to college

42% were working

92% were working or going to college

to put this into terms of this section

" 68% ~~went to~~ going to college" = probability of randomly selecting a student and then going to college is 0.68

$P(C) = 0.68$
$P(W) = 0.42$
$P(C \cup W) = .92$

What percent not going to college nor working...

$P((C \cup W)') = 1 - P(C \cup W) = 1 - 0.92 = 0.08$

8% not working nor going to college

Working and College?

$P(C \cap W) = P(C) + P(W) - P(C \cup W)$
$.68 + 0.42 - 0.92 = 0.18$

18%