Warm up

- Simplify $\dfrac{7!}{3!\,4!} = \dfrac{7\cdot\cancel{6}\cdot 5\cdot\cancel{4}\cdot\cancel{3}\cdot\cancel{2}\cdot\cancel{1}}{\cancel{3}\cdot 2\cdot 1\cdot\cancel{4}\cdot\cancel{3}\cdot\cancel{2}\cdot\cancel{1}} = 7\cdot 5 = 35$

- A community service project asks a group of 10 people for ....

  (a) A supervisor and a driver (different people)
  (b) 3 volunteers (for an unrelated project)

(a) How many ways to assign a supervisor and a driver? 10

(b) How many ways to choose 3 volunteers?

(a) Is it enough to know which people were chosen? No, we need to know who and what we chose them for $\Rightarrow$ sets aren't enough, ordered lists $\Rightarrow$ permutation

$$P(10,2) = 10\cdot 9 = \frac{10!}{(10-2)!}$$

(b) Enough to know who was chosen? Yes! Sets are enough $\Rightarrow$ combinations

$$C(10,3) = \frac{P(10,3)}{3!} = \frac{10!}{(10-3)!\,3!} = \frac{10\cdot 9\cdot 8}{3\cdot 2}$$

27 Mar

Suppose we were again making playlists, say 10 songs long, with 15 to choose from.

But, we could use the same song in it multiple times.

How would things change?

Step 1: Choose 1st song: 15 options
Step 2: Choose 2nd song: 15 options
Step 3: 3rd song: 15 options
⋮
Step 10: 10th song: 15 options

We would have

$$\underbrace{15 \cdot 15 \cdot 15 \cdot \cdots \cdot 15}_{10} = 15^{10}$$

What's different from this and permutations?

Here we allowed repetition.

The number of ordered lists that are k long from a total n elements allowing repetition* is

$$n^k$$

* with repetition means repeated as many times

10 people applied for 4 different positions and a single person can be hired for multiple positions.

How many possible different hirings?

→ is knowing which people were hired enough?
No, we need who and what, means sets are not enough ⇒ order matters

→ is repitition allowed? 'A single person ... for multiple positions' means repitition is allowed.

$$10^4$$

4 ← the number of position fillings

10 ↗ number of people available

$$= 10 \cdot 10 \cdot 10 \cdot 10 = 10{,}000$$

How many ordered outcomes of flipping a coin 6 times are there?

This one doesn't look exactly like the previous. But consider a potential outcome,

H T T T H T

Every outcome has the form

_ _ _ _ _ _

where each blank is either H or T

So each coin flipping outcome is a 6 long list of 2 elements with repetition allowed.

$$2^{6} = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 64$$

6 ← length of list

number of elements

What about our 2-topping pizzas if we allowed using the same topping twice?
(toppings M, H, P, S, B)

if order mattered then $\underline{5}\ \underline{5}$ have $5^2 = 25$

MM	MH HM	MP PM	MS SM	MB BM
HH	HP PH	HS SH	HB BH	
PP	PS SP	PB BP		
SS	SB BS			
BB				

Let's circle the pizzas that are actually different.

15 actually different pizzas

Before $C(n,k) = \frac{P(n,k)}{k!}$

but here $15 \neq \frac{5^2}{2!} = \frac{25}{2} = 12.5$

well before we divided by 2! because each circle had 2 ordered pizzas in it.

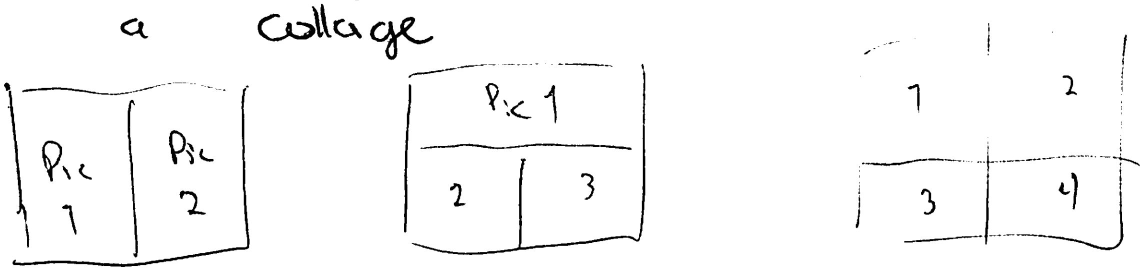
But we can say every 2-topping pizza has

2 different toppings or 1 doubled topping

$C(5,2) = \frac{P(5,2)}{2!} = \frac{5 \cdot 4}{2} = 10$ + 5 $\Rightarrow 15$

Sometimes we might need to break the problem into easier to count pieces.

A picture collage app takes 2-4 photos and makes a collage



You want to make collages of your 5 cats and 1 portrait for each cat. You don't want to reuse pictures in a collage.

How many collages possible?

→ is knowing which cat pictures enough?
no, we need to know order too So lists not sets

→ is repetition allowed?
no, we don't want to reuse pictures

permutations
each collage uses

2 photos 3 photos or 4 photos

$$P(5,2) + P(5,3) + P(5,4)$$

$$\frac{5!}{(5-2)!}$$

$$5\cdot 4 + 5\cdot 4\cdot 3 + 5\cdot 4\cdot 3\cdot 2$$

$$20 \qquad 60 \qquad 120 \qquad = 200$$

7.1

In 6.1 we introduced sets
6.2 counting sets with $\cup$, $\cap$, complements
6.3 general way to count scenarios
6.4 ways to count special scenarios (perms & combin)

7 starts to combine these
Relate it to probability

7.1 will help introduce the language of doing all this

In the context of probability, we often deal with the probability of some ~~exp~~ particular <u>outcome</u> of some <u>experiment</u>

The set of all outcomes of an experiment is the <u>sample space</u>.

- if our experiment was to flip 3 coins...
an example outcome is TTH.

Our sample space would be (it would $2^3 = 8$)

$$S = \{ HHH, \ HTH, \ THH, \ TTH, \ HHT, \ HTT, \ THT, \ TTT \}$$

If our experiment was to select someone currently on the ISS

an example outcome is Anne M.

Our sample space would be

$$S = \{\text{David S, Anne M, Oleg K, Oleg S, Nick H, Christina K}\}$$

If our ~~sample space~~ experiment was select 2-topping pizzas (with the usual 5 toppings)...

Our sample space has $C(5,2) = \frac{5 \cdot 4}{2} = 10$ outcomes

If we looked at vegetarian pizzas $C(3,2) = 3$

$V = \{MS, MB, SB\}$ and $V \subseteq S$

Given a sample space S, an event E is a subset of S. We call the outcomes in an event the favorable outcomes.

We say an event E occurs in an experiment if E contains at least 1 outcome.

Consider the experiment of flipping a coin and then rolling a die.

an exampe outcome: T3, H6

The event of ~~rolling~~ flipping a heads is

$H = \{H1, H2, H3, H4, H5, H6\}$

The event of rolling an even #

$E = \{H2, H4, H6, T2, T4, T6\}$

How is $E \cap H$ described as an event?

$E \cap H$ is the event of rolling an even number and fipping heads

Is $E \cap H$ feasible (does it occur)?

Yes, $E \cap H = \{H2, H4, H6\}$ $\quad n(E \cap H) > 0$

What is $(E \cap H)'$ described as an event?

The event of not rolling an even and flipping heads at the same time.

Is $(E \cap H)'$ feasible (does it occur)?

$n((E \cap H)') = \underset{12}{n(S)} - \underset{3}{n(E \cap H)} = 9$

So $(E \cap H)'$ is feasible

Consider experiment of rolling a red die and then a blue die.

an example outcome: 32 note how this is different than 23

What is the event of the two dice rolls summing to 7?

$V = \{16, 25, 34, 43, 52, 61\}$

What is the event of rolling two even #s?

$E = \{22, 24, 26, 42, 44, 46, 62, 64, 66\}$

Interpret $V \cap E$ as an event.
The set of outcomes of dice results summing to 7 and both #'s even.

So $V \cap E = \emptyset$, $n(V \cap E) = 0$

So $V \cap E$ is not feasible, it does not occur.

What is $n(S) = 6 \cdot 6 = 36$