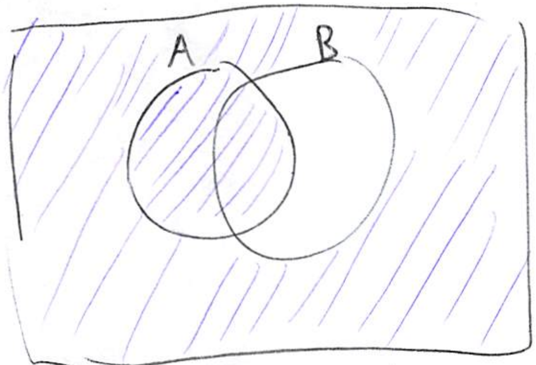
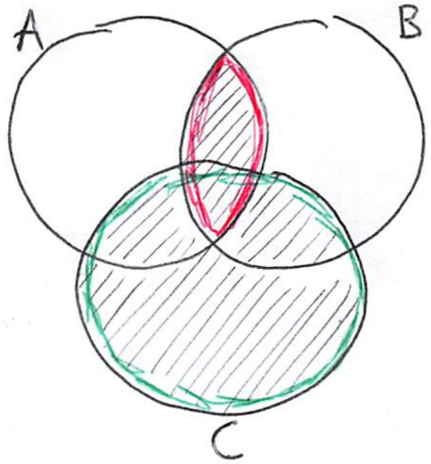
Draw a Venn diagram and shade the region corresponding to $A \cup B'$



union
everything in A or B'

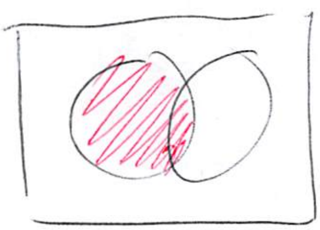
B' or complement of B is everything not in B

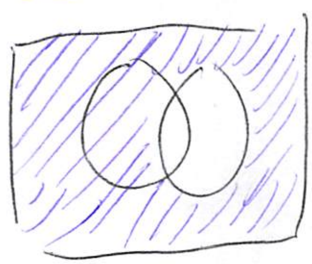
Use $\cup$, $\cap$, $'$s to describe



$(A \cap B) \cup C$

$A \cup B'$





6.2 Cardinality

20 Mar

A very useful property of a set is its size, ie how many elements it contains.
We call this the <u>cardinality</u> of a set, denoted by $n(S)$ for some set S.

A natural question is to ask is how does the cardinality change in relation to our operations?

$$n(A \cup B), \quad n(A \cap B), \quad n(A'), \quad n(A \times B)$$

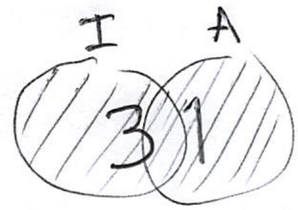
union, intersect, comp., cart. prod.

Suppose a class survey found that 31 students participated in either IM events or official school teams.

$n(I \cup A) = 31$

$I = \{$ students that do IM stuff $\}$

$A = \{$ student athletes $\}$

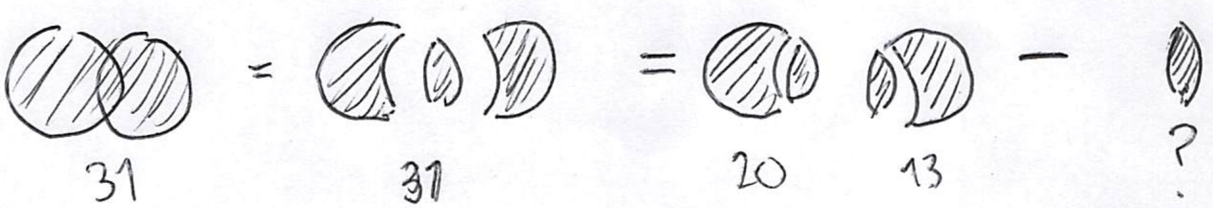


What if it also told us that
20 students play in IM events
13 students are student athletes

$n(I) = 20$

$n(A) = 13$

Why doesn't $20 + 13 = 31$?



so it's not $n(I) + n(A) = n(I \cup A)$ X

$$n(I \cup A) = n(I) + n(A) - n(I \cap A)$$

This means we have information to find $n(I \cap A)$, to find # of students who do IM and school teams

rearranged... $n(I \cap A) = n(I) + n(A) - n(I \cup A)$

$20 \quad 13 \quad - 31$

$= 2$

2 students do both.

By rearranging we find:

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

all of cone from

$$n(A \cap B) = n(A) + n(B) - n(A \cup B)$$

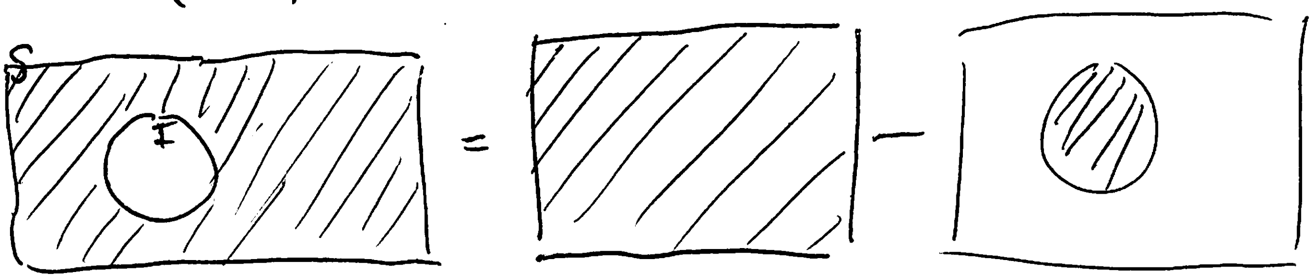
$$n(A) = n(A \cup B) - n(B) + n(A \cap B)$$

$$n(B) = n(A \cup B) - n(A) + n(A \cap B)$$

we said 20 IM students, suppose we knew that 90 students were surveyed.

How many students don't participate in IM?

↳ $n(I')$



$n(S)$ $n(I)$

$90 - 20$

$n(I') = 70$

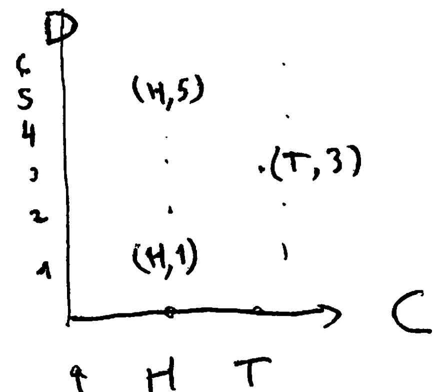
$n(A') = n(S) - n(A)$

Lastly we have $n(A \times B)$

$C \times D$ is outcomes of flipping a coin then rolling a die

$C = \{\text{Heads}, \text{Tails}\}$

$D = \{1, 2, 3, 4, 5, 6\}$



How many points in ?

for each coin flip there's $n(D)$ many options for its 2nd coord

So $n(C \times D) = n(C) * n(D)$
$= 2 \cdot 6 = 12$

In fact $n(A \times B \times C \times \dots \times Z) = n(A) * n(B) * \dots * n(Z)$

6.3

Suppose for some sets you were told

$$n(A \cup B) = n(A) + n(B)$$

What would we know?

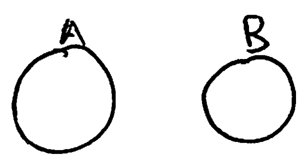
in general $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

so here $n(A \cap B) = 0$

If $n(A \cap B) = 0$, ie $A \cap B = \emptyset$, we say

A and B are <u>disjoint</u>

if $n(A \cap B) = 0$ then our venn diagram looks



Many real life questions about sizes of sets deal with unions of disjoint sets and cartesian products of sets

Keeping track of sizes of combinations of the above involve adding sizes of sets and multiplying sizes of sets

Knowing when to do what comes from understanding the <u>additive principle</u> and <u>multiplicative principle</u>

ex At a restaurant, there are 9 ~~options~~ entrees with meat and 2 vegetarian options.

How many main entrees are there?

$$n(M \cup V) \quad \text{where} \quad \begin{array}{l} M = \{\text{meat entrees}\} \\ V = \{\text{nonmeat entrees}\} \end{array}$$

M and V are disjoint, so

$$n(M \cup V) = n(M) + n(V) - \overset{0}{n(M \cap V)}$$
$$9 + 2 = 11$$

For dessert, there are 5 flavors of ice cream and 3 sizes of each one available.

How many desserts can be ordered?

each dessert is formed by choosing a flavor and choosing a size, that's an ordered pair, so we're asking

$$n(F \times S) \quad \text{where} \quad \begin{array}{l} F = \{\text{flavors}\} \\ S = \{\text{sizes}\} \end{array}$$

$$n(F \times S) = n(F)\,n(S)$$
$$5 \cdot 3 = 15 \checkmark$$

How about ways to order a main meal and a dessert?

An order consists of choice of entree and choice of dessert, that's an ordered pair.

We're asking

$$n(M \times D) \qquad M = \{\text{entrees}\}, \quad D = \{\text{desserts}\}$$

$$n(M \times D) = n(M) * n(D)$$

$$11 \cdot 15 = 165$$

Are there other ways to get these answers?

Yes, one is called a <u>decision algorithm</u>.

How would we have built an order at the restaurant?

- Our first <u>step</u> would be to choose a meal:
  there were 2 options

  9+2 =11
  - Alternative 1: w/ meat 9 options
  - Alternative 2: w/o meat 2 options

- Our second <u>step</u>, dessert:

  5·3 =15
  - First step of building dessert: flavor 5 options
  - 2nd step of : size 3 options

11* 15 =165

DA:
- start at most indent layers and work outwards
- <u>add alternatives</u> and <u>multiply steps</u>

$(9+2) * (5 \cdot 3) = 165$

An art project allows students to make a painting with one of the 3 color schemes:

- only 1 color
- a primary color and a secondary
- a primary, secondary, ternary color

There are only 4 colors to choose from

What questions would someone trying to pick a project ask? Which color scheme will I use?
What colors will I use on that?

- alternative 1: only 1 color
  4 (step 1: choose primary color 4 options

- alternative 2: a primary + secondary color
  4·3 =12 (step 1: choose primary: 4 options
  step 2: choose secondary: 3 options

- alt. 3: a prim, sec, tern colors
  4·3·2 =24 (Step 1: choose primary: 4 options
  step 2: secondary: 3
  step 3: ternary: 2

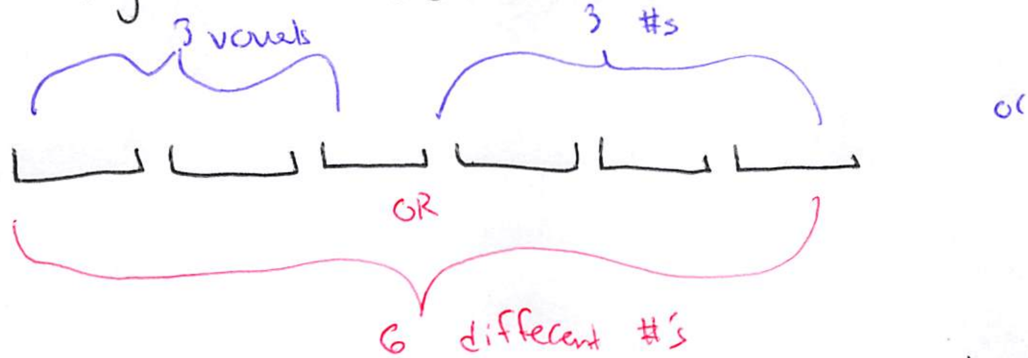
4 + 12 + 24 = 40

steps get multiplied

$(4) + (4 \cdot 3) + (4 \cdot 3 \cdot 2)$

alternates get added

A lottery ticket can be filled out either by listing 3 vowels and then 3 numbers or by listing 6 different numbers.



What questions would someone filling out a ticket ask?

- am I going to do vowels & #'s or just #'s?
- what #'s / vowels would I choose?

alt 1 3 vow, 3 #'s

Step 1 choose vowel 1 : 5
5
⋮ 5
10
10
Step 6 10

alt 2 6 diff #'s

Step 1 10
9
⋮ 8
7
6
steps 6 5