Warmup

How many ways are there to arrange 3 books on a bookshelf?

How do we know we got them all?
How does it generalize?

Chapter 6 is about counting things... 3/18

- if there's 20 pizza toppings, how many 2-topping pizza's are there?
- How many ways are there to fill in a March Madness bracket?
- How many ways to shuffle a 30 song playlist?

To address questions like these and many more it's useful to understand <u>sets</u>.

A <u>set</u> is a collection of items, referred to as elements of a set.

We typically use curly brackets, $\{\ \}$ to denote a set.

$W=\{1,2,3\}$ W is the set that contains 1, 2, and 3.

We use the symbol $\in$ to denote an item being an element of a set

Amazon $\in$ {Google, Amazon, Facebook}

The set X is a subset of Y ($X \subseteq Y$)
if
every element of X is an element of Y.

($X \subseteq Y$ if $x \in X$ implies $x \in Y$)

If two sets have exactly the same elements, we say they are equal.

ex List the elements of the following sets:

- A the set of integers from 0 to 5 — $A = \{0, 1, 2, 3, 4, 5\}$
- B is the set of suits in a deck of cards — $B = \{$hearts, spades, diamonds, clubs$\}$
- C the of black suits in a deck of cards — $C = \{$spades, clubs$\}$

Which of the following are true?

$A \subseteq B$? no, $0 \in A$ and $0 \notin B$

$B \subseteq C$? no, hearts $\in B$ but hearts $\notin C$

$C \subseteq B$? yes, everything in C lives in B

$B = C$? nope, see

$C \in B$? no,

$\{1,2\} \in \{\{1\}, \{1,2\}, \{1,2,3\}\}$

$2 \notin$

The set containing no elements has a special name and symbol, the <u>empty set</u>

$$\emptyset = \{ \} \leftarrow \text{the set containing nothing}$$

note: if A is a set, then $\emptyset \subseteq A$

We can define a set by describing its elements instead of listing them. There exists a standard notation for this.

$$B = \{ n \mid \text{conditions } n \text{ must satisfy to be in } B \}$$

$\uparrow$ some typically or general element of the set

ex

$$B = \{ x \mid x \text{ is an integer and } |x| < 4 \}$$

$$= \{3, -3, 2, -2, 1, -1, 0\} = \{-3, -2, -1, 0, 1, 2, 3\}$$

$$D = \{ x \mid x < 10 \text{ and } x > 15 \} = \emptyset$$

$C = \{ (d_1, d_2) \mid d_1$ and d_2 are the results of ~~[illegible]~~ rolling 2 distinguishable dice $\}$

$$
\begin{aligned}
C = \{ & (1,1), (1,2), (1,3), \ldots, (1,6), \\
& (2,1), \ldots \qquad (2,6), \\
& \vdots \\
& (6,1), \ldots \qquad (6,6) \}
\end{aligned}
$$

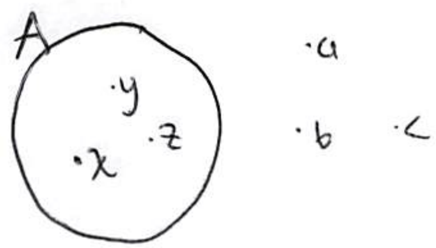
$D = \{ x \mid x$ is the sum of 2 dice rolls $\}$

$$
\begin{array}{lllll}
2, & 3, & 4, & \ldots & 7 \\
3, & 4, & & \ldots & 8 \\
\vdots & & & & \vdots \\
7 & & \ldots & & 12
\end{array}
$$

$D = \{2, 3, 4, \ldots, 12\}$

One way to help visualize sets are Venn Diagrams.

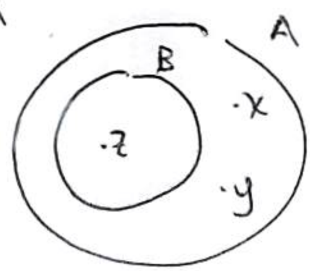
We can draw a set A as a circle and draw elements of A as points inside the circle. Things not in A as points outside.



$x, y, z \in A$

$a, b, c \notin A$

If $B \subseteq A$ then we can draw B as a circle fully contained by A



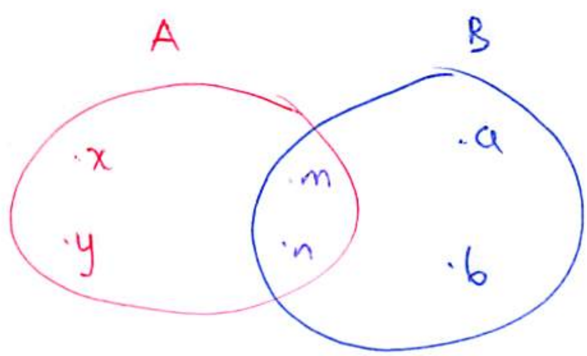
$x \in A$

$z \in A$

$z \in B$

$x \notin B$

Consider



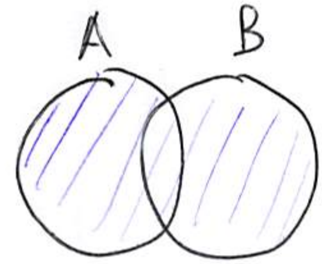
Here $A \nsubseteq B$ and $B \nsubseteq A$

- The set of things in either A or B is $\{x, y, a, b, m, n\}$
  ↳ we call this $A \cup B$ (A union B)

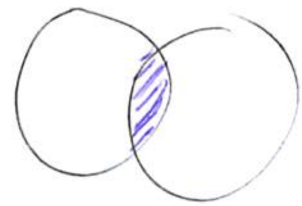
- The set of things in both A and B is $\{m, n\}$
  ↳ we call this $A \cap B$ (A intersect B)

Pictorially

$A \cup B$ is



$A \cap B$ is



ex

Apples, Bananas, and Oranges are fruit available at food lion, F. Apples, Bananas, and Kiwis are fruit available at Harris Teeter, H.

- What are the fruits available at a grocery store?

  $F \cup H = \{$apples, bananas, oranges, kiwi$\}$

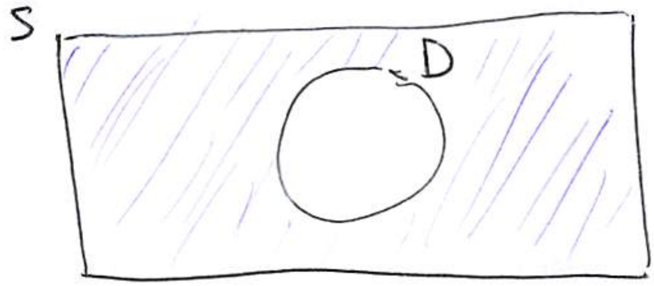
- What are the fruit I can get at any grocery store?
  (only looking at fruit available at both stores)

  $F \cap H = \{$apples, bananas$\}$

Another way to make a new set given D is to look at everything not in D.
This is the complement of D, D'.

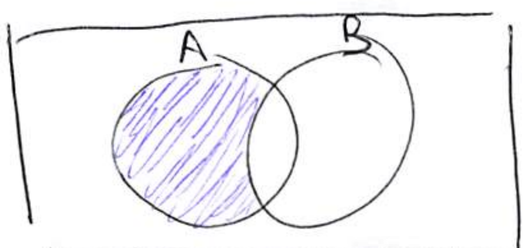
Often we declare some sample space S for which $D \subseteq S$ and then D' is everything not in D but in S.

D'



Using these three operators, we can concisely describe each region of a Venn Diagram

- things in A but not in B



$A \cap (B')$

- things in A or B but <u>not</u> in both

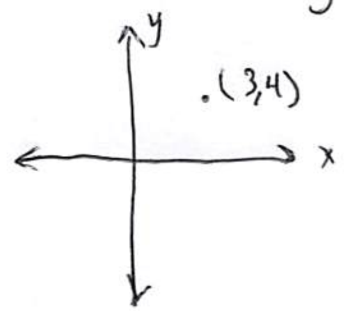
$(A \cup B) \cap ((A \cap B)')$

- ~~things in A and B but~~ not

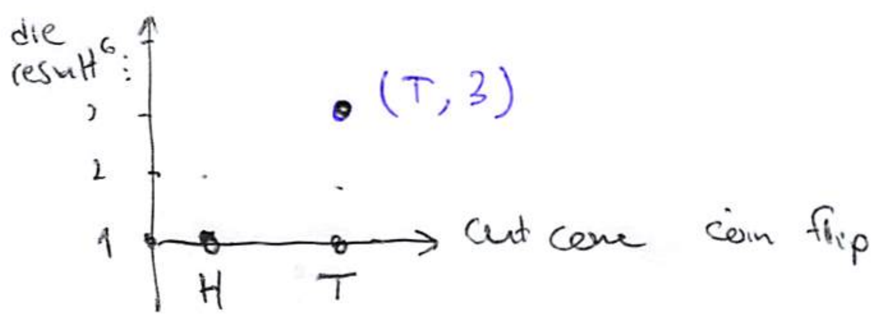
The last operator we'll mention is the Cartesian Product, where given sets A and B the cartesian product $A \times B$ is the set of ordered pairs (a, b) where $a \in A$, $b \in B$.

Concisely $A \times B = \{ (a,b) \mid a \in A \text{ and } b \in B \}$

ex the xy-plane is the cartesian product, $\mathbb{R} \times \mathbb{R}$ where $\mathbb{R}$ is the set of real numbers



Consider the outcomes of flipping a coin then rolling a die.



$$\text{Coin Flip} \times \text{Die Result} = \{ (H,1), (H,2), \ldots, (H,6), (T,1), \ldots (T,6) \}$$

Exam 2

- average ~ 75
  median ~ 82

- if ≤ 4 absences at end of semester
  lowest exam replaced with best exam

WebAssign

- 30% of total grade
  ~ 13 %
  → every webassign assignment not done
  knocks >1% off final grade

if you have questions:
reach out to me, Erica, MMC