Refresher

Find the inverse of

$A = \begin{pmatrix} 5 & -1 \\ 1 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 3 \\ 4 & 12 \end{pmatrix}$

if $M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ then $M^{-1} = \frac{1}{ad-bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

* if $ad-bc = 0$, then M has no inverse

$$A^{-1} = \frac{1}{5 \cdot 2 - (-1) \cdot 1} \begin{pmatrix} 2 & 1 \\ -1 & 5 \end{pmatrix} = \begin{pmatrix} 2/11 & 1/11 \\ -1/11 & 5/11 \end{pmatrix}$$

$B^{-1} = \frac{1}{12 - 12} \leftarrow 0$ So B is non invertible

27 Feb

- exam on 6th Mar
  covering: 4.3, 4.4, 5.1, 5.2 (syllabus will be updated to reflect this)
- Similar review material will be made available
- The Sunday after exam 2, hw for 5.1 and 5.2 are due. That is the Sunday before Spring break
  ☆ start early, use it as review for the exam ☆

5.2

A linear programming (LP) problem, in 2 unknowns, is finding pts (x, y) that satisfy given constraints of the form

$$a_1 x + b_1 y \geq c_1$$
$$a_2 x + b_2 y \geq c_2$$
$$\vdots$$

that either maximize or minimize (as specified by a given problem) a given objective function

$$px + qy$$

In the bakery problem of last class, the constraints were

$$3C + 1B \leq 25$$
$$1C + 2B \leq 20$$
$$B \geq 0$$
$$C \geq 0$$

and the objective function was $9C + 20B$

At the time, we didn't prove or explain why, but $(0,10)$, making 0 cookies, 10 brownies, maximized the objective function, $6 \cdot 0 + 20 \cdot 10 = 200$.

Here the <u>optimal value</u> was 200.
and the <u>optimal solution</u> was $(0,10)$.

For a given objective function, there is only one (if it exists) optimal value. <u>but</u> there can be multiple optimal solutions.

Fundamental Theorem of Linear Programming

- if an LP problem has optimal solutions, then one of them is a corner of feasible region
- LP problems with a bounded feasible region <u>always</u> have optimal solution

Immediate consequence: How to solve a LP problem

1. determine the feasible region of the given constraints
2. determine the corners of ↗
3. * if bounded * plug each corner into the obj function
4. The optimal value is the min/max (depends on the problem) of the values from step 3

If unbounded, we just need to do a little more ex A student field trip is being planned. The two available vans can fit a total of 12 people. To receive permission for trip, there must be at least 2 adults. To receive school funding the # of students cannot be less than half the # of adults.

Q1: If it cost \$10 for adults and \$5 for students, what's the cheapest field trip?

Q2: What's the maximum # of students that can be brought?

Q3: If adults can carry 4 snack each and students can carry 1, what's the max # of snacks that can be brought?

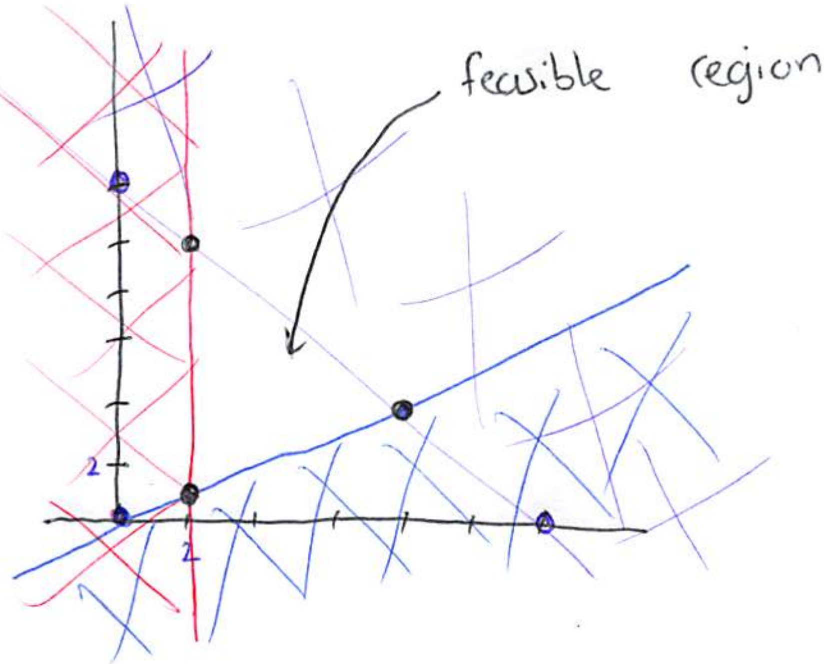
First let's assign variable:

$x \rightarrow$ # adults $\qquad y \rightarrow$ # students

Let's find constraints:

- "... vans can fit a total of 12 people" $\quad x + y \leq 12$
- "...at least 2 adults" $\quad x \geq 2$
- "# students cannot be less than ½ # adults..." $\quad y \geq \frac{1}{2}x$

not < means ≥



What are our corners?

intersections of... $x+y=12$ and $x=2$ $\Rightarrow$ $(2)+y=12$ so $y=10$

$(2, 10)$

$x+y=12$ $\quad$ $y=\frac{1}{2}x$ $\Rightarrow$ $(2y)+y=12$ $\quad$ $y=4$

$2y=x$ $\longrightarrow$ $2(4)=x$ $\quad$ $x=8$

$(8,4)$

$x=2$ $\quad$ $y=\frac{1}{2}x$ $\Rightarrow$ $y=\frac{1}{2}(2)$ $\quad$ $y=1$

$(2, 1)$

- The feasible region is bounded, so by FTLP, the optimal solution for each Q, is one of these 3 corners.
- What was objective function of Q_1, Q_2, Q_3?

OF_1 : $10x+5y$, min

OF_2 : y, max

OF_3 : $4x+1y$, max

Corners	$10x+5y$, min	y, max	$4x+1y$, max
$(2,10)$	$10(2)+5(10)=70$	$\boxed{10}$	18
$(8,4)$	$10(8)+5(4)=120$	4	$\boxed{36}$
$(2,1)$	$10(2)+5\cdot(1)=\boxed{25}$	1	9

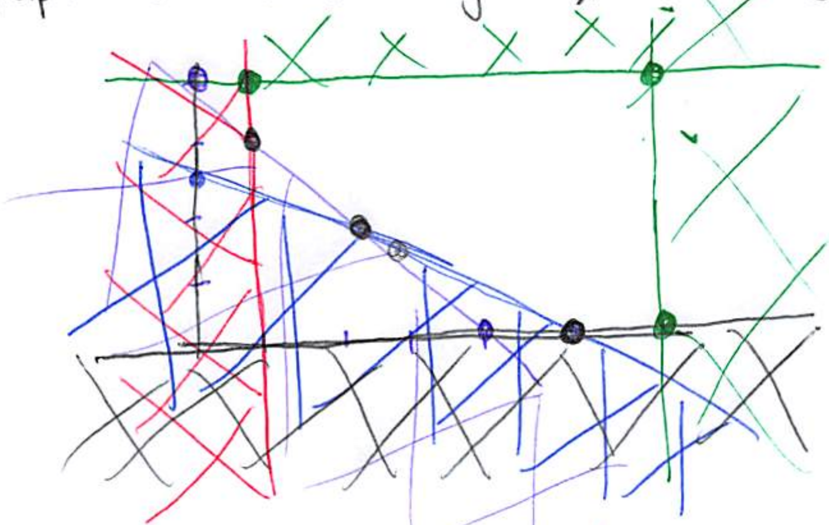
- The cheapest is \$25, by bringing 2 adults 1 student
- The max # of students that can be brought is 10, by bringing 2 adults 10 students
- The max # of snacks that can be brought is 36, by bringing 8 adults and 4 students

What if feasible region is unbounded?
- Introduce artificial constraints that make it bounded
- this would add new corners, keep track of original corners and new corners
- FTLP applies to this new bounded region
- if the max/min comes from an original corner, that corner gives the optimal value
- if the max/min comes from a new corner, the optimal value is <u>unbounded</u>, no optimal solutions exist

ex Suppose given $y \geq 0$
$x + y \geq 4$
$x \geq 1$
$x + 2y \geq 5$

with objective functions: minimize $20x + 30y$
maximize $-4000x + 2000y$

graph the feasible region & find corners



* unbounded feasible region *

→ $(1,3)$ $(6,4)$
→ $(3,1)$ $(6,0)$
→ $(5,0)$ $(1,4)$

the largest x value of corners is 5, choose something bigger $x \leq 6$
the largest y value is 3, choose something bigger $y \leq 4$

corners	min, $20x+30y$	max $-4000x+2000y$
$(1, 3)$	110	2000
$(3, 1)$	$\boxed{90}$ original corner, optimal value	-10000
$(5, 0)$	100	-20000
$(6, 4)$	240	-16000
$(6, 0)$	120	$\boxed{4000}$ new corner, OV is unbounded
$(1, 4)$	140	-24,000