Refresher

Solve the following game (find optimal strategy of row player, col player, and expected value of the game)

$$\begin{array}{c|c} & \begin{array}{ccc} x & y & z \end{array} \\ \begin{array}{c} a \\ b \\ c \end{array} & \begin{pmatrix} 5 & -1 & 0 \\ 3 & -3 & 0 \\ 2 & 0 & 5 \end{pmatrix} \end{array}$$

reduction by dominance

- row dominates another if it is entry wise larger
- col dominates another if it is entry wise smaller

$$\begin{array}{cc} & \begin{array}{ccc} 2 & & 3 \end{array} \\ \begin{array}{c} 4 \\ 1 \\ \end{array} & \begin{pmatrix} 5 & -1 & 0 \\ 3 & -3 & 0 \\ 2 & 0 & 5 \end{pmatrix} \end{array}$$

- row player plays c
- col player plays y
- EV of G is 0

look for a saddle pt

- row minima
- col maxima

$$\begin{pmatrix} 5 & \boxed{-1} & 0 \\ 3 & \boxed{-3} & 0 \\ 2 & \boxed{0} & 5 \end{pmatrix}$$

↑ saddle point

5.1 Linear Inequalities 2/25

One batch of cookies req. 3 cups of flour and 1 cup of sugar. One batches of brownies req 1 cup of flour and 2 cups sugar.
You have 25 cups of flour and 20 cups of sugar.
If a batch cookies sells for \$9 and a batch of brownies sells for \$20, how much should get made to max revenue?

- One idea would be to use all flour and all sugar

$$25 = 3C + 1B$$
$$20 = 1C + 2B$$

$\Rightarrow$ leads to 6 cookies, 7 batches brownies which would make $9 \cdot 6 + 20 \cdot 7 = 54 + 140 = 194$

<u>but</u> we can make 10 batches of brownies and no cookies

$$3(0) + 1(10) = 10 \text{ cups of flour}$$
$$1(0) + 2(10) = 20 \text{ cups of sugar}$$

we'd still have 15 cups of flour, but we'd earn $9 \cdot 0 + 20 \cdot 10 = 200$

Here the requirement of using all our resources did not find max revinue.

The problem stated says we can use at most 25 cups flour, 20 cups sugar

$$3C + 1B \leq 25$$
$$1C + 2B \leq 20$$

We want to check all (C, B) that satisfy to see what maximizes $9 \cdot C + 20 \cdot B$

This is called linear programming.

We need to understand inequalities.

$a \leq b$ means a is less than or equal to b

$a \geq b$ means a is greater than or equal to b

We can manipulate inequalities as follows:

1. We can add a quantity to both sides — if $x \leq y$ then $x + a \leq y + a$

2. We can multiply or divide both sides by a nonnegative # — if $x \leq y$ and $c \geq 0$ then $cx \leq cy$

3. We can multiply or divide both sides by a negative # <u>but</u> we must reverse the inequality — if $x \leq y$ and $c < 0$ then $cx \geq cy$

4. We can reverse both sides and the inequality — if $x \leq y$ then $y \geq x$

ex

- if $3x+5y \geq 0$
- then by add $-3x$ to both sides: $5y \geq -3x$
- $5 \geq 0$, we can divide by 5: $y \geq \frac{-3}{5}x$

- $-5x \geq 10$
- $-5 < 0$, we can divide by -5, but we must reverse the sign: $x \leq 10/_{-5}$

$x \leq -2$

(can check, $-3 \leq -2$ so $(-5)(-3) \geq 10$, $15 \geq 10$ ✓)

A <u>linear inequality</u> is an expression where if we treated it as an equality (change $\leq$ or $\geq$ to $=$) it would be a linear equality.

$x + 5y + 10z \geq -7$ is a linear inequality

$y \geq x^2 + 2$ is <u>not</u> a linear inequality

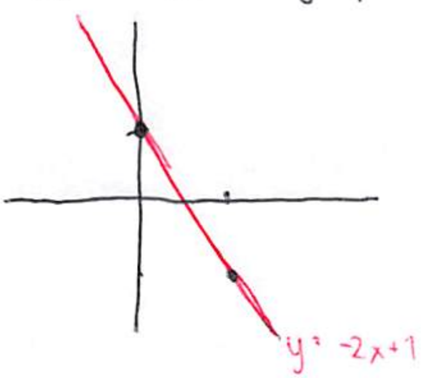
Graphing a linear inequality

We can graph the solution set of a linear inequality, this is also called the feasible region.

ex graph $2x+y \geq 1$

- note: $2x+y=1$ then $2x+y \geq 1$

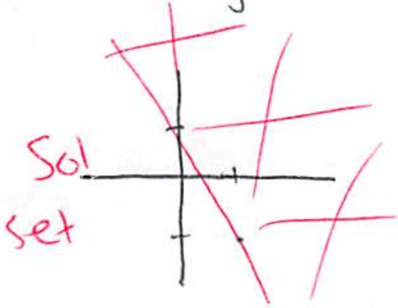
↳ so first graph $2x+y=1 \Rightarrow y=-2x+1$



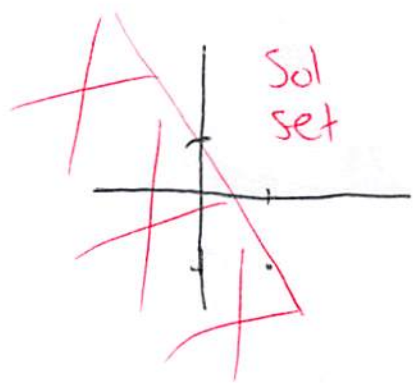
- but there are a lot more points satisfying $2x+y \geq 1$ than $2x+y=1$

☆ the inequality $ax+by \geq c$ is a half plane with $ax+by=c$ being the dividing line.

So $2x+y \geq 1$ is either



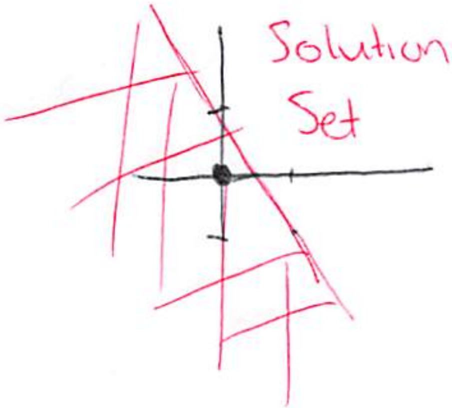
OR



* where we X'd out the region not in the solution set ☆

How do we determine which half plane?

↳ Choose a point on one side & check if it satisfies our inequality.



does $(0,0)$ satisfy
$$2x + y \geq 1 \ ?$$

$$2 \cdot 0 + 0 = 0$$
$$0 \not\geq 1$$

So $(0,0)$ is <u>not</u> in the solution set X

The solution set of a system of inequalities is the set of points that satisfy all of the inequalities in the system.

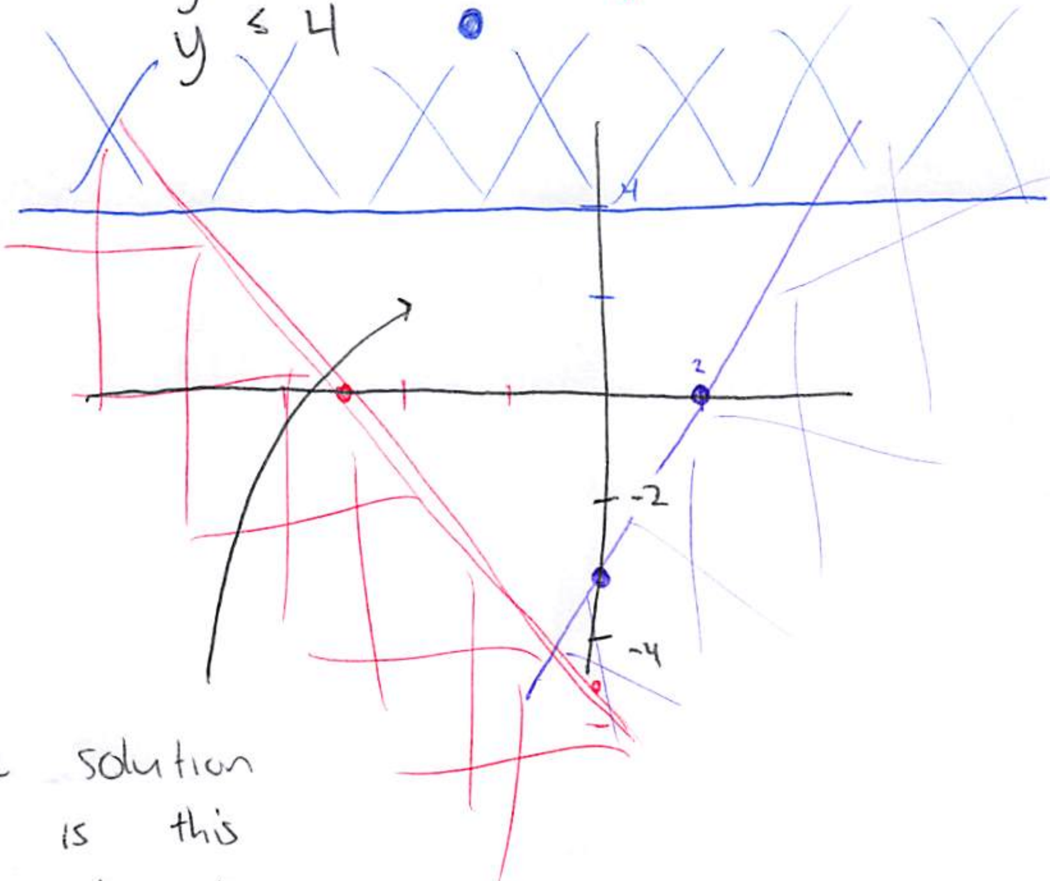
Graphically, it's the unshaded region after we've shaded the half plane that doesn't satisfy a given inequality, for each inequality.

ex

$3x - 2y \le 6$ → $-2y \le 6 - 3x$ → $y \ge -3 + \frac{3}{2}x$

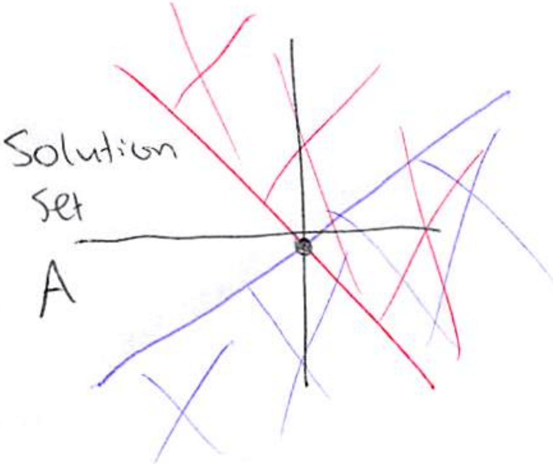
$x + y \ge -5$ → $y \ge -x - 5$

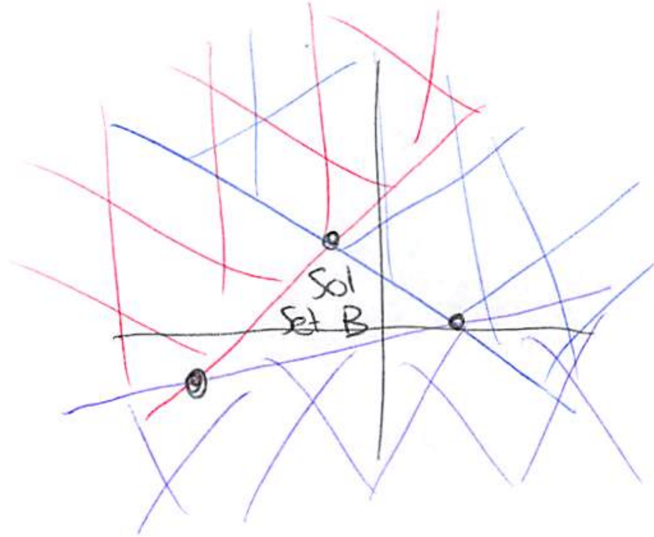
$y \le 4$



the solution set is this inside triangle

This is also our first example of a bounded solution set





The sol. set A can be extended infinitely for some direction, thus it is <u>unbounded</u>.

The sol. set B. is completely enclosed, thus it is <u>bounded</u>.

In both, there are <u>corners</u> of the solution set (the • dots in the above). If we treat the inequalities as equalities and find their intersection, we find potential corners.

What are te corners of

$$\begin{aligned} 3x - 2y &\leq 6 \\ x + y &\geq -5 \\ y &\leq 4 \end{aligned} \quad ?$$

corner 1: where does $3x - 2y = 6$ and $x + y = -5$ intersect?

$x + y = -5$

$y = -x - 5$ substitute $\rightarrow$ $3x - 2(-x - 5) = 6$

$3x + 10 + 2x = 6$

$5x = -4$

$\leftarrow$ $x = -4/5$

$y = -\left(\frac{-4}{5}\right) - 5 = -21/5$ $\quad$ $(-4/5, -21/5)$

corner 2: $3x - 2y = 6$ and $y = 4$

... $(22/3, 4)$

corner 3: $x + y \geq -5$ and $y = 4$

... $(-9, 4)$

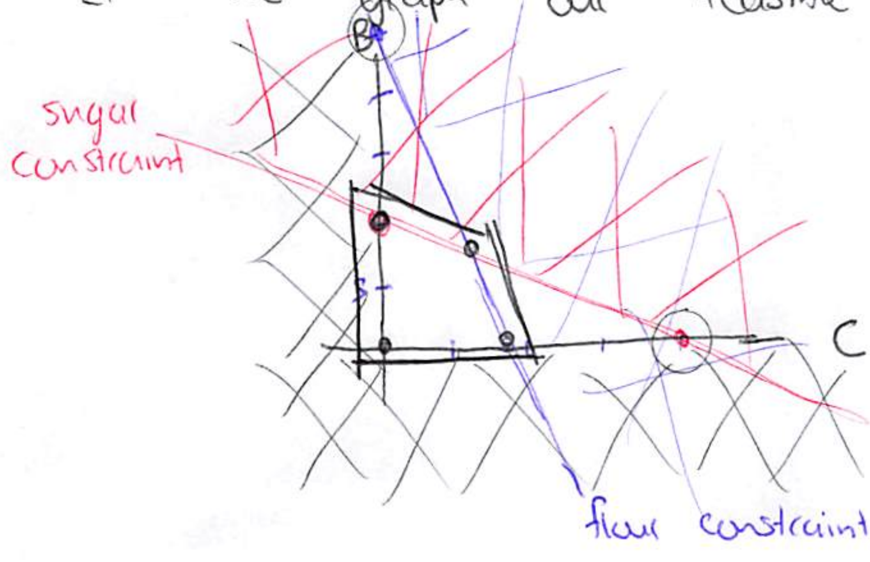
Back to bakery ...

	cookies	brownies	available
flour	3	1	25
sugar	1	2	20

The feasible amount of brownies and cookies we can make satisfy

$$3C + 1B \leq 25$$
$$1C + 2B \leq 20$$
$$B \geq 0$$
$$C \geq 0$$

If we graph our feasible region



note: $3C + 1B = 25$ intersect $B = 0$ is not a corner
$1C + 2B = 20$ intersect $C = 0$ is not a corner

For systems of 2 variables inequalities containing 4 or more ineq's will always have "corners" that are not corners of the feasible set.

We will use the graph as our tool for determining which are contributing corners.

Note smaller systems with ~~every~~ intersections that aren't corners are still possible.

$2x - y \leq 2$

$y + 1 \geq x$

$x \leq 5$