Solve the following:

$$2(A^T + B)$$

where $A = \begin{bmatrix} 2 & 3 & 1 \\ 0 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 1 \\ -2 & 2 \\ 0 & 3 \end{bmatrix}$.

$A^T = \begin{bmatrix} 2 & 0 \\ 3 & 1 \\ 1 & 2 \end{bmatrix}$, $A^T + B = \begin{bmatrix} 2 & 0 \\ 3 & 1 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 1 \\ -2 & 2 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 3 \\ 1 & 5 \end{bmatrix}$

$$2(A^T + B) = 2\begin{bmatrix} 1 & 1 \\ 1 & 3 \\ 1 & 5 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 2 & 6 \\ 2 & 10 \end{bmatrix}$$

30 Jan

- Exam next Wednesday

- still need some blue books
- final details about Exam told on Monday
- Goal for studying:
  - understand all webassign problems
  - understand all problems introduced in lectures
    - ↳ be able to follow each step
    - ↳ aim to know why each step is done

Matrix Multiplication

Recall AB not always defined.

If A is an $m \times n$ matrix and B is a $p \times q$ matrix AB is defined if $n=p$

$$AB$$ is a $m \times q$ matrix

$(m \times n)\ (p \times q)$

$n = p$ defined

m, q as the new dimensions

ex

Let A be 3×4 and B be 4×2

Is AB defined?

$(3\times4)\ (4\times2)$, $=$

So yes AB is defined, it has dimensions 3×2.

Is BA defined?

$(4\times2)\ (3\times4)$, $\neq$

So no BA is not defined

☆ Order matters here ☆

ex is AA^T defined?

$(m\times n)\ (n\times m)$, $=$ ✓

Yes, if A was $m\times n$, AA^T is $m\times m$

is A^TA defined?

$(n\times m)\ (m\times n)$, $=$ ✓

Yes, A^TA $n\times n$

if $m\neq n$, $A=\begin{bmatrix}2 & 3 & 1\\ 0 & 1 & 2\end{bmatrix}$ then $AA^T\neq A^TA$

In general $AB\neq BA$

Simpler Case:

If A was a row vector with dimension $1\times m$ and B was a $m\times 1$ col vector. AB would be defined and have dimension 1×1

But we can treat 1×1 matrices exactly as numbers.

To get this number, we will mutiply entries in A (from left to right) with entries in B (from top to bottom) and add them all together.

ex

$$\begin{bmatrix} 5 & 3 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix} = \underbrace{5*1}_{5} + \underbrace{3*2}_{6} + \underbrace{2*4}_{8} = 19$$

$$\begin{bmatrix} 2 & 3 \end{bmatrix} \begin{bmatrix} 7 \\ 1 \end{bmatrix} = 2*7 + 3*1 = 14 + 3 = 17$$

$$\begin{bmatrix} 1 & 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 5 \\ 8 \\ 0 \\ 2 \end{bmatrix} = \underbrace{1\cdot 5}_{5} + \underbrace{1\cdot 8}_{8} + \underbrace{2\cdot 0}_{0} + \underbrace{3\cdot 2}_{6} = 19$$

Matrix multiplication is defined as follows:

The ij-th entry of AB is the result of multiplying the i-th row of A with the j-th col of B.

ex

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} * \begin{pmatrix} 6 & 2 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} (1\ 2)\begin{pmatrix} 6 \\ 0 \end{pmatrix} & (1\ 2)\begin{pmatrix} 2 \\ 1 \end{pmatrix} \\ (3\ 4)\begin{pmatrix} 6 \\ 0 \end{pmatrix} & (3\ 4)\begin{pmatrix} 2 \\ 1 \end{pmatrix} \end{pmatrix} = \begin{pmatrix} 6 & 4 \\ 18 & 10 \end{pmatrix}$$

2×2 $\qquad$ 2×2

✓

new dimen

the 1,1 entry is $(1\ \ 2)\begin{pmatrix} 6 \\ 0 \end{pmatrix} = 1 \cdot 6 + 2 \cdot 0 = 6$

1,2 $\qquad (1\ \ 2)\begin{pmatrix} 2 \\ 1 \end{pmatrix} = 1 \cdot 2 + 2 \cdot 1 = 4$

2,1 $\qquad (3\ \ 4)\begin{pmatrix} 6 \\ 0 \end{pmatrix} = 3 \cdot 6 + 4 \cdot 0 = 18$

2,2 $\qquad (3\ \ 4)\begin{pmatrix} 2 \\ 1 \end{pmatrix} = 3 \cdot 2 + 4 \cdot 1 = 10$

ex $A = \begin{pmatrix} 1 & 2 \\ 0 & 1 \\ -1 & 3 \end{pmatrix}$ $\quad B = \begin{pmatrix} 5 & 7 \\ 0 & 4 \end{pmatrix}$

$3 \times 2 \qquad 2 \times 2 \qquad$ AB defined, 3×2

$$\begin{pmatrix} 1 & 2 \\ 0 & 1 \\ -1 & 3 \end{pmatrix}\begin{pmatrix} 5 & 7 \\ 0 & 4 \end{pmatrix} = \begin{pmatrix} 5 & 15 \\ 0 & 4 \\ -5 & 5 \end{pmatrix}$$

$(AB)_{11} \quad \begin{pmatrix} 1 & 2 \end{pmatrix}\begin{pmatrix} 5 \\ 0 \end{pmatrix} = 1\cdot 5 + 2\cdot 0 = 5$

$(AB)_{12} \quad \begin{pmatrix} 1 & 2 \end{pmatrix}\begin{pmatrix} 7 \\ 4 \end{pmatrix} = 1\cdot 7 + 2\cdot 4 = 15$

$(AB)_{21} \quad \begin{pmatrix} 0 & 1 \end{pmatrix}\begin{pmatrix} 5 \\ 0 \end{pmatrix} = 0\cdot 5 + 1\cdot 0 = 0$

$(AB)_{22} \quad \begin{pmatrix} 0 & 1 \end{pmatrix}\begin{pmatrix} 7 \\ 4 \end{pmatrix} = 0\cdot 7 + 1\cdot 4 = 4$

$(AB)_{31} \quad = -5$

$(AB)_{32} \quad = 5$

BA defined? $\quad (2 \times \underbrace{2) \cdot (3}_{\neq} \times 2)$ $\quad$ not defined ex

	people who read the book	people who saw the movie
Lord of the Rings	100	350
Princess Bride	125	250
To Kill a Mockingbird	200	150

Suppose 20% of people who read the book own the book.

Suppose 5% of people who saw the movie own the movie.

How many own the book or own movie?

↳ well for each story

(# who read it)*0.20 + (# who watched it)*0.05

$$\underset{(3\times 2)}{\begin{bmatrix} 100 & 350 \\ 125 & 250 \\ 200 & 150 \end{bmatrix}} \underset{(2\times 1)}{\begin{bmatrix} 0.20 \\ 0.05 \end{bmatrix}} = \underset{(3\times 1)}{\begin{bmatrix} 37.5 \\ 37.5 \\ 47.5 \end{bmatrix}}$$

What about just the book?

$$\begin{bmatrix} 100 & 350 \\ 125 & 250 \\ 200 & 150 \end{bmatrix} \begin{bmatrix} 0.20 \\ 0 \end{bmatrix} = \begin{bmatrix} 20 \\ 25 \\ 40 \end{bmatrix}$$

Just the move?

$$\begin{bmatrix} 100 & 350 \\ 125 & 250 \\ 200 & 150 \end{bmatrix} \begin{bmatrix} 0 \\ 0.05 \end{bmatrix} = \begin{bmatrix} 17.5 \\ 12.5 \\ 7.5 \end{bmatrix}$$

We can get all of these at once

$$\underset{(3\times 2)}{\begin{bmatrix} 100 & 350 \\ 125 & 250 \\ 200 & 150 \end{bmatrix}} \underset{(2\times 2)}{\begin{bmatrix} 0.20 & 0 \\ 0 & 0.05 \end{bmatrix}} = \underset{(3\times 2)}{\begin{bmatrix} 20 & 17.5 \\ 25 & 12.5 \\ 40 & 7.5 \end{bmatrix}}$$

Recall 1 times any number is just that number.

Is there some similar '1' matrix? <u>Yes</u>, we

call it the <u>identity matrix</u>.

The identity matrix, I, is
- square
- has 1's on the diagonal
- has 0's everywhere else

<u>ex</u>

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1\cdot 1 + 2\cdot 0 & 1\cdot 0 + 2\cdot 1 \\ 3\cdot 1 + 4\cdot 0 & 3\cdot 0 + 4\cdot 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

Question? Given a matrix A can we find a matrix B such that

$$AB = I \ ?$$

Sometimes. ~~[illegible]~~
Here B would be called the <u>inverse</u> of A or A^{-1}

But before talking about that let's understand matrices and variables.

ex What is $\begin{bmatrix} 5 & 2 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 5x + 2y + 3z$

so

$$5x + 2y + 3z = 7 \iff \begin{bmatrix} 5 & 2 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 7$$

This works for systems of equations too!

$$\begin{aligned} 5x + 2y + 3z &= 7 \\ 6y + z &= 0 \\ -x + -3y + 3z &= 1 \end{aligned} \iff \underbrace{\begin{bmatrix} 5 & 2 & 3 \\ 0 & 6 & 1 \\ -1 & -3 & 3 \end{bmatrix}}_{A}\underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} 7 \\ 0 \\ 1 \end{bmatrix}}_{B}$$

Now our system looks like

$$AX = B$$

How would we solve

$$ax = b$$

if a, b were numbers?

- divide by a
- multiply by a's inverse

We can't divide by A but if A^{-1} ($A^{-1}A = 1$) then

$$AX = B$$
$$\downarrow$$
$$A^{-1}AX = A^{-1}B$$
$$\downarrow$$
$$IX = A^{-1}B$$
$$\downarrow$$
$$X = A^{-1}B$$

multiply both sides by A inverse

In one sense, this is exactly what we've been doing with reducing the augmented matrix of a system. But remember this method didn't always give a unique solution

↳ if the inconsistent system (ie no solutions) then there's no A^{-1}

↳ if redundant system (ie infinite solution) then there's no A^{-1}