28 Jan

ex

In July 2011 Apple stock increased from \$350 to \$400 per share and Google stock increased from \$500 to \$600 a share. If you invested \$22000 in these stocks at the beginning of the month and sold them for \$26000 at the end of the month, how many stocks of each had been purchased? (set up the augmented matrix that could be used to solve this)

- assign variables: A be for Apple Stock
  G Google Stock

- interpret data/constraints

beginning of month → $22000 = 350A + 500G$

end of month → $26000 = 400A + 600G$

$$\left(\begin{array}{ccc} 350 & 500 & 22000 \\ 400 & 600 & 26000 \end{array}\right)$$

Comment about WebAssign:

Solutions to

$$y = 3x + 6 \quad \rightarrow \text{ slope is } 3$$
$$2y = 6x + 12 \quad \rightarrow \text{ slope is } 3$$

if these were 2 different lines, then no solution

these could be the same line

$$(x, 3x+6)$$

Recall

An $m \times n$ matrix is an array / table of (in this class numbers) with m rows and n columns.

ex $\begin{pmatrix} 350 & 500 & 22000 \\ 400 & 600 & 26000 \end{pmatrix}$ has 2 rows
3 col's

it is 2×3 matrix

We can reference an entry of a matrix by specifying the row and column it lives in.

$$M = \begin{bmatrix} 4 & 8 & 15 \\ 16 & 23 & 42 \end{bmatrix}$$

$M_{1,2}$ (the element in the 1st row and 2nd column)

$M_{12} = 8$

$M_{2,1}$ (the element in 2nd row, 1st col)

$M_{2,1} = 16$

In general, the entries of a 3×2 matrix are

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$$

for an $m \times n$ matrix

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & & a_{2n} \\ \vdots & & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

a_{ij} lives in the i th row, j th column ex

$A = \begin{bmatrix} x+1 & y \\ x+y & -y \end{bmatrix}$

- if $a_{11} = 2 \Rightarrow x+1 = a_{11} = 2$
  $x = 1$

- if $a_{21} = 3 \Rightarrow x+y = a_{21} = 3$
  $x+y=3$
  $(1)+y=3$
  $y=2$

So $A = \begin{bmatrix} 2 & 2 \\ 3 & -2 \end{bmatrix}$

Some dimensions give matrices special names

- if a matrix has 1 row and multiple columns we call it a row matrix or row vector

ex: $A = \begin{bmatrix} 1 & 3 & 5 & 1 & 0 \end{bmatrix}$ is a 1×5 row matrix

- if a matrix has 1 col and multiple rows we call it a column matrix or column vector

ex $B = \begin{bmatrix} 2 \\ 3 \\ 12 \end{bmatrix}$ is a 3×1 col matrix

- if a matrix has equal number of rows and columns, it is a square matrix

ex $C = \begin{bmatrix} 1 & 5 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$ is a 3×3 square matrix

We say two matrices are <u>equal</u> if

- they have the same dimensions
- the corresponding entries are equal

if $A = B$ then $a_{ij} = b_{ij}$ for all rows i and columns j

<u>ex</u>

if $A = \begin{bmatrix} 5x & -z \\ 3x+y & y \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 7 \\ 10 & 7 \end{bmatrix}$

and $A = B$. What do we know?

↳ $a_{11} = b_{11}$ so $5x = 5 \Rightarrow x = 1$

↳ $a_{12} = b_{12}$ so $-z = 7 \Rightarrow z = -7$

↳ $a_{21} = b_{21}$ $\quad 3x + y = 10 \Rightarrow 3(1) + (7) = 10$

↳ $a_{22} = b_{22}$ $\quad y = 7 \Rightarrow y = 7$

Here a single equation with matrices 'contained' 4 equations.

☆ we can use matrices to concisely represent data and equations ☆

Addition & Subtraction

If two matrices have the same dimensions then we can define addition and subtraction between them by adding or subtract their corresponding entries.

ex $\begin{bmatrix} 5 & 6 \\ 10 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 5+1 & 6+1 \\ 10+2 & 1+3 \end{bmatrix} = \begin{bmatrix} 6 & 7 \\ 12 & 4 \end{bmatrix}$

$$\begin{bmatrix} x & y \\ z & w \end{bmatrix} + \begin{bmatrix} x & 2 \\ y & 7 \end{bmatrix} = \begin{bmatrix} x+x & y+2 \\ z+y & w+7 \end{bmatrix} = \begin{bmatrix} 2x & y+2 \\ z+y & w+7 \end{bmatrix}$$

Using our entry coordinate notation, we can say

$$(A+B)_{ij} = A_{ij} + B_{ij}$$

'the element in the i^{th} row j^{th} column of $A+B$ is the element in the i^{th} row j^{th} column of A plus the element in the i^{th} row j^{th} column of B'

ex $\begin{bmatrix} 2 & 3 \\ 5 & 6 \\ 1 & 1 \end{bmatrix} - \begin{bmatrix} 10 & 5 \\ 8 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2-10 & 3-5 \\ 5-8 & 6-1 \\ 1-0 & 1-1 \end{bmatrix} = \begin{bmatrix} -8 & -2 \\ -3 & 5 \\ 1 & 0 \end{bmatrix}$

ex A parent company oversees 3 smaller business. The first has monthly revenue function $100x$ and cost function $45x + 1500$. The 2nd has rev $110x$ and cost $55x + 1200$. The third has rev $90x$ and cost $45x + 1000$. Construct a Rev, Cost, and Profit matrix.

$$Rev = \begin{bmatrix} 100x \\ 110x \\ 90x \end{bmatrix} \qquad Cost = \begin{bmatrix} 45x + 1500 \\ 55x + 1200 \\ 45x + 1000 \end{bmatrix}$$

Profit = Rev - Cost

$$Profit = \begin{bmatrix} 100x \\ 110x \\ 90x \end{bmatrix} - \begin{bmatrix} 45x + 1500 \\ 55x + 1200 \\ 45x + 1000 \end{bmatrix} = \begin{bmatrix} 55x - 1500 \\ 55x - 1200 \\ 45x - 1000 \end{bmatrix}$$

Scalar Multliplication:

We can multiply matrices by a number c by multiplying ever entry by c.

Symbolically $(cA)_{ij} = c(A_{ij})$

ex
$$5\begin{bmatrix} 10 & 12 \\ 13 & -1 \end{bmatrix} = \begin{bmatrix} 5 \cdot 10 & 5 \cdot 12 \\ 5 \cdot 13 & 5 \cdot (-1) \end{bmatrix} = \begin{bmatrix} 50 & 60 \\ 65 & -5 \end{bmatrix}$$

Now we can really make familiar equations

ex Profit we just said $\begin{bmatrix} 55x - 1500 \\ 55x - 1200 \\ 45x - 1000 \end{bmatrix}$

$$= \begin{bmatrix} 55x \\ 55x \\ 45x \end{bmatrix} - \begin{bmatrix} 1500 \\ 1200 \\ 1000 \end{bmatrix} = \begin{bmatrix} 55 \\ 55 \\ 45 \end{bmatrix} x - \begin{bmatrix} 1500 \\ 1200 \\ 1000 \end{bmatrix}$$

It turns out that matrices behave 'nicely',
ie

$$A + (B + C) = (A + B) + C$$
$$A + B = B + A$$
$$A + \mathbf{0} = A = \mathbf{0} + A$$
$$A + (-A) = \mathbf{0} = (-A) + A$$

$$c(A + B) = cA + cB$$
$$(a+b)A = aA + bA$$
$$1 \cdot A = A$$
$$0 \cdot A = \mathbf{0}$$

$\mathbf{0}$ here is the 'zero matrix', it has 0 for every entry

ex $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

If they behave like numbers in some ways, what are ways they differ?

Transposition:

The transpose of a matrix M, signified by a superscript T, M^T, is a new matrix where rows and columns have been swapped

$$(M^T)_{ij} = M_{ji}$$

ex

$$M = \begin{bmatrix} 2 & 3 \\ 1 & 4 \\ 0 & -1 \end{bmatrix}$$

M is a 3×2 matrix

M^T is a 2×3 matrix

$$M^T = \begin{bmatrix} 2 & 1 & 0 \\ 3 & 4 & -1 \end{bmatrix}$$

Here $(A+B)^T = A^T + B^T$

$$(A^T)^T = A$$

ex

$$M = \begin{bmatrix} 1 & 2 \\ x & 7 \\ -1 & y \end{bmatrix} \qquad M^T = \begin{bmatrix} 1 & 6 & x+y \\ 2 & 7 & y \end{bmatrix}$$

find x, y

- $M = (M^T)^T = \left(\begin{bmatrix} 1 & 6 & x+y \\ 2 & 7 & y \end{bmatrix}\right)^T = \begin{bmatrix} 1 & 2 \\ 6 & 7 \\ x+y & y \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ x & 7 \\ -1 & y \end{bmatrix}$

so $x = 6$

$x + y = -1$

$(6) + y = -1 \Rightarrow y = -7$

<u>ex</u>

$A = \begin{bmatrix} 1 & 3 \\ 5 & 7 \end{bmatrix}$ $\quad$ A is 2×2 ✓

A^T $\quad 2\times2$

$A^T = \begin{bmatrix} 1 & 5 \\ 3 & 7 \end{bmatrix}$

does there exist $A = A^T$ yea; symmetric

Matrix Multiplication:

Let A and B be matrices. Again

$$A * B$$

is not always defined. Here AB is defined only if the number of columns in A equal the number of rows of B

$$\underset{m\times n}{A} * \underset{p\times q}{B}$$

only if $n = p$