Refresher / Throwback:

Where does the line $y = 4x + 5$ intersect the line parallel to $y = -x + 2$ that goes through $(3,3)$?

- let's find an equation for 2nd line:
  - in general $y = mx + b$
  - is parallel to $y = -x + 2$ so $m = -1$
  - goes through $(3,3)$ so

$$(3) = -1 * (3) + b$$
$$3 = -3 + b$$
$$b = 6$$

→ $y = -x + 6$

- let's find intersection:
  - one way is to set both lines equal

$$4x + 5 = -x + 6$$
$$5x = 1$$
$$x = 1/5$$

  - plug in this x to one of our lines

$$-\left(\frac{1}{5}\right) + 6 = \frac{29}{5} = 5.8$$

- The lines intersect at $(1/5, 29/5)$

Systems as Matrices

- introduce associating systems with matrices and vice versa
- an algorithm (Gauss-Jordan Elimination) to 'solve' these matrices

- It is important (could be on tests) to know how to express a system w/ a matrix and to interpret the output of G.J.E.

- It is useful (not tested) to see and understand G.J.E. for systems of 3 or more equations

* for webassign 3.2 technology can be used on any problem

From system to matrix

Suppose we had the system

$$5x + 10y = 4$$
$$x - y = -3$$

in eq1: 5 and 10 are coeff. of x and y resp.

eq2: 1 and -1 " "

$$\begin{array}{c|c} & \begin{array}{cc} x & y \end{array} \\ \begin{array}{c} eq^1 \\ eq^2 \end{array} & \begin{bmatrix} 5 & 10 \\ 1 & -1 \end{bmatrix} \end{array}$$

this is the <u>coefficient matrix</u>

If we add an additional col corresponding to the Right Hand Side of our equations we get

$$\begin{bmatrix} 5 & 10 & 4 \\ 1 & -1 & -3 \end{bmatrix}$$

this is the <u>augmented matrix</u>

<u>ex</u> Let $\begin{bmatrix} 1 & 3 & 1 \\ 2 & 1 & 2 \end{bmatrix}$ be the augmented matrix of some system, what is it?

- # rows is # of eqs
- # cols -1 is # of variables

$$\underline{1}\,x + \underline{3}\,y = \underline{1}$$
$$\underline{2}\,x + \underline{1}\,y = \underline{2}$$

ex Same question but fix

$$\begin{array}{c} \\ eq_1 \\ eq_2 \\ eq_3 \end{array}\begin{array}{c} \begin{array}{cccc} a & b & c & RHS \end{array} \\ \left[\begin{array}{cccc} 1 & 0 & 1 & 1 \\ 2 & 1 & 3 & 4 \\ -7 & \frac{1}{2} & 2 & 2 \end{array}\right] \end{array}$$

$$1 * a + 0 \cdot b + 1\, c = 1$$
$$2a + 1 \cdot b + 3 \cdot c = 4$$
$$-7a + \tfrac{1}{2} \cdot b + 2 \cdot c = 2$$

What can we do with our matrix?

* imp idea: certain operations don't change the solution *

These operations expressed in terms of aug. matrices are:

1. replace a row R_i by $a * R_i$ $(a \neq 0)$
2. replace a row R_i by $aR_i \pm bR_j$ $(a \neq 0)$
3. switch order of rows

ex

type ①
$$\begin{bmatrix} 0 & 1 & 4 \\ 1 & -3 & 2 \end{bmatrix} \xrightarrow{R_1 \to 3*R_1} \begin{bmatrix} 0 & 3 & 12 \\ 1 & -3 & 2 \end{bmatrix}$$

②
$$\begin{bmatrix} 0 & 3 & 12 \\ 1 & -3 & 2 \end{bmatrix} \xrightarrow{R_2 \to R_1 + R_2} \begin{bmatrix} 0 & 3 & 12 \\ 1 & 0 & 14 \end{bmatrix}$$

③
$$\begin{bmatrix} 0 & 3 & 12 \\ 1 & 0 & 14 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{bmatrix} 1 & 0 & 14 \\ 0 & 3 & 12 \end{bmatrix}$$

what's the system associated to

$$\begin{aligned} x + 0y &= 14 \\ 0x + 3y &= 12 \end{aligned} \qquad \begin{aligned} x &= 14 \\ y &= 4 \end{aligned}$$

so this is also a solution to

$$\begin{aligned} y &= 4 \\ x - 3y &= 2 \end{aligned}$$

So row operations can transform one matrix to an easy to solve one.

easy to solve form looks like:

$$\begin{bmatrix} 1 & 0 & a \\ 0 & 1 & b \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \end{bmatrix}, \begin{bmatrix} 1 & & \text{\huge 0} & a_1 \\ & \ddots & & a_2 \\ \text{\huge 0} & & 1 & \vdots \\ & & & a_n \end{bmatrix}$$

* key point: 1's along diagonal; 0's above and below the ones

Gauss-Jordan Reduction brings a matrix into this form (if it is possible)

GJR:

Let $i=1$

- make the i^{th} diag entry 1
- make the entries above and below the i^{th} diag 0. (don't rearrange rows before the i^{th} row)
- repeat with $i+1$

Short ex:

$$\begin{bmatrix} 3 & 15 & -3 \\ 1 & 8 & -4 \end{bmatrix}$$

Step 1: try to make 1st diag 1

$R_1 \to \frac{1}{3} R_1$

$$\begin{bmatrix} 1 & 5 & -1 \\ 1 & 8 & -4 \end{bmatrix}$$

Step 1b: make entries below 1st diag 0

$R_2 \to R_2 - R_1$

$$\begin{bmatrix} 1 & 5 & -1 \\ \overset{1-1=}{0} & \overset{8-5=}{3} & \overset{-4--1}{-3} \end{bmatrix}$$

Step 1: done

Step 2: make 2nd diag 1

$R_2 \to \frac{1}{3} R_2$

$$\begin{bmatrix} 1 & 5 & -1 \\ 0 & 1 & -1 \end{bmatrix}$$

Step 2b: make entry above 2nd diag 0

$R_1 \to R_1 - 5R_2$

$$\begin{bmatrix} 1 & 0 & \overset{-1-\cancel{5}(-1)}{4} \\ 0 & 1 & -1 \end{bmatrix}$$

Step 2: done

GJR: done

System for this aug matrix?

$1x + 0y = 4$

$0x + 1y = -1$

so $x = 1$

$y = -1$

So $x = 1$, $y = 1$ is solution

$3x + 15y = -3$

$x + 8y = -4$

When can't it be put into this 'nice' form?

- we could get a row of all 0's except in last col.

ex
$$\begin{array}{cccc} x & y & z & \\ \end{array}$$
$$\begin{pmatrix} 1 & 0 & 0 & 2 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 1 & 4 \end{pmatrix} \iff \begin{aligned} x &= 2 \\ 0 &= 3 \\ z &= 4 \end{aligned}$$

that's never true so no solution

↳ pts to an inconsistent system

- we could get an all 0's row

$$\begin{pmatrix} 1 & 0 & 2 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 0 \end{pmatrix} \iff \begin{aligned} x + 2z &= 3 \\ y &= 2 \\ 0 &= 0 \end{aligned}$$

$\Rightarrow$ is true; no unique solution

↳ infinite solutions; redundant system

question: is $\begin{pmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \end{pmatrix}$ problematic?

$$\iff \begin{aligned} x &= 2 \\ y &= 3 \\ z &= 0 \end{aligned}$$

perfectly fair

So if we have tech to solve these our problem is thus to set up the equations

A group of 33 people want to travel in cars that seat 4 people, cars that seat 5, and vans that seat 8. There are 6 people who will drive. We have 2 people that will drive vans. No vehicle will be driven with an empty seat. How many 4-seat cars, 5-seat cars, and vans will be used?

① assign variable to the unknowns being asked for

- U for foUr seater
- I for fIve seater
- V for Vans

② what do we know / what is required of us

→ everyone must be in a vehicle

$$33 = 4U + 5I + 8V$$

→ $$6 = U + I + V$$

→ 2 vans get used

$$V = 2$$

③ Make Matrix & solve

$$4u + 5I + 8v = 33$$
$$u + I + v = 6$$
$$v = 2$$

$$\begin{array}{cccc} u & I & v & \\ \left[\begin{array}{cccc} 4 & 5 & 8 & 33 \\ 1 & 1 & 1 & 6 \\ 0 & 0 & 1 & 2 \end{array}\right] \end{array} \rightarrow \begin{array}{cccc} u & I & v & \\ \left[\begin{array}{cccc} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{array}\right] \end{array}$$

4 interpret:

$u = 3$	3 four seaters
$I = 1$	1 five seater
$v = 2$	2 vans

~5) check

You've mixed a few decks of cards together and then removed some. The # of hearts equals the # of spades and is twice the # of clubs. The number of clubs and diamonds is 40. There are 14 more red cards than black. How many of each suit?

① assign variables to our unknowns:

S #of spades
C clubs
D diamonds
H hearts

② what do we know / what are our constraints?

"# of hearts equals # of spades" → $H = S$

"... and is twice # of clubs" → $H = 2C$

"# clubs ~~equ~~ and # diamonds is 40" → $C + D = 40$

"there are 14 more red than black" $H + D = S + C + 14$

③

$$\begin{array}{c} \begin{array}{ccccc} S & C & D & H & \end{array} \\ \left(\begin{array}{ccccc} -1 & 0 & 0 & 1 & 0 \\ 0 & -2 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 40 \\ -1 & -1 & 1 & 1 & 14 \end{array}\right) \end{array} \rightarrow$$