As a passenger in a car on the highway you keep track of elapsed minutes and mile markers. You collect enough data to make the following table:

time (hours)	0	0.5	1.5
mile marker	20	47	101

On the way back you do something similar

time	0	1	1.5
mile m.	143	107	72

Can either be modeled by a linear function? If so what is it? There was traffic on ~~the~~ one of trips; justify on which trip you think it happened.

- linear function: constant rate of change
- $\frac{\Delta m}{\Delta h}$ $\frac{47-20}{0.5-0} = \frac{27}{0.5} = 54$ ✓ $\Rightarrow$ $M(h) = 54 \cdot h + 20$

$\frac{101-47}{1.5-0.5} = \frac{54}{1} = 54$

· return: $\frac{\Delta m}{\Delta h}$ $\frac{107-143}{1-0} = \frac{-36}{1} = -36$ X

$\frac{72-107}{1.5-1} = \frac{-35}{0.5} = -70$

traffic?: maybe trip 2

- next class : Wed 23rd
  - ↳ first class with attendance (making seating chart)
  - ↳ last class of passing in blue books for extra credit

3.1

Systems of equations

- Recall that $y = mx + b$ is an equation that we can interpret as a function

- Now let's look at it as an equation. A more general form of a <u>linear equation of 2 unknowns</u> is

$$ax + by = c$$

(here 'b' is not to be interpreted as 'y-intercept')

<u>def</u> A <u>solution</u> of an equation is a pair of values that satisfy the equation. (when we 'plug in' those values the expression is true)

<u>ex</u> $5x + 3y = 2$

$(-2, 4)$ or $x = -2, y = 4$ is a solution because

$5*(-2) + 3*(4) = 2$

$-10 + 12 = 2$

$(2 = 2$ ✓$)$

non ex

$(1,2)$ is <u>not</u> a solution

$5*(1) + 3(2) = 5 + 6 = 11 \neq 2$

Solutions might not be unique

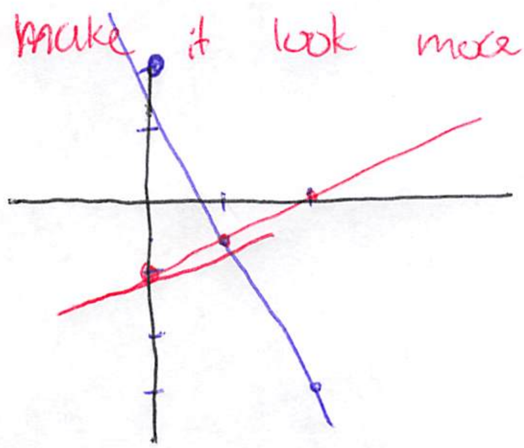
A single linear equation has infinitely many solutions.

If we were to plot the solutions of an equation onto a graph we would have the same line as if we graphed the equation treated as a function

ie we already know how to plot solutions of a linear equation!

ex: graph the solution set of $5x+2y=4$ and the solution set of $x-2y=2$.

- make it look more familiar:



$$5x+2y=4$$
$$2y=-5x+4$$
$$y=-\tfrac{5}{2}x+2$$

$$x-2y=2$$
$$-2y=-x+2$$
$$y=\tfrac{x}{2}-1$$

$(1,-0.5)$ is a solution to both $5x+2y=4$ and $x-2y=2$

$(1,-0.5)$ is a solution to the system

$$5x+2y=4$$
$$x-2y=2$$

A system of two equations of two unknowns looks like

$$ax + by = c$$
$$dx + ey = f$$

(where a, b, c, d, e, f are numbers not necessarily all different)

A solution to a system of equations are values that are a solution to each of the equations in the system.

ex

In our previous system:

$(1, -0.5)$ was a solution

$(0, 2)$ was not, (it isn't solution to $x - 2y = 2$)

We found this answer graphically before, but we can do it algebraically.

review: if $a = b$ then $a + c = b + c$

if $a = b$ then $ac = bc$

if $a = b$ and $c = d$ then $a + c = b + d$

ex

$$5x + 2y = 4$$
$$x - 2y = 2$$

$\xrightarrow{eq2 + eq1}$

$$(5x + 2y) + (x - 2y) = 4 + 2$$
$$6x + \cancel{0y} = 6$$
$$x = 1$$

• let's plug in $x = 1$ into one of our eq's

$5 * 1 + 2y = 4 \Rightarrow 5 + 2y = 4$

$$2y = -1$$
$$y = -1/2$$

ex find solution to:

$$3x + 2y = 1$$
$$6x - 5y = 5$$

- in the last one, we just added them and the y's canceled, here we might need to modify it beforehand.

- $$\begin{aligned} 3x + 2y &= 1 \\ 6x - 5y &= 5 \end{aligned} \xrightarrow{-2 * eq1} \begin{aligned} -6x - 4y &= -2 \\ 6x - 5y &= 5 \end{aligned}$$

let's add $eq1 \xrightarrow{+} eq2$

$$(-6x - 4y) + (6x - 5y) = -2 + 5$$
$$-9y = 3$$
$$y = -1/3$$

- plug in $y = -1/3$ to an equation:

$$3x + 2 * (-1/3) = 1$$
$$3x - 2/3 = 1$$
$$3x = 5/3$$
$$x = 5/9$$

Do two lines always intersect? <u>No</u>

Therefore, not all sytems have solutions. If a system has no solutions we call it <u>inconsistent</u>

$2x + y = 4$
$4x + 2y = -3$

I claim is inconsitent

$2x + y = 4 \Rightarrow y = -2x + 4$
$4x + 2y = -3 \Rightarrow y = -2x - 3/2$

same slope $\therefore$ parallel
$\therefore$ no solution

$-2 \cdot eq1 \rightarrow$ $-4x - 2y = -8$
$4x + 2y = -3$

adding eq1 + eq2 $\quad 0x + 0y = -11$

but $0 \neq -11$

There can also be infinite solutions (we call such a system <u>redundant</u>)

$2x + y = 4$
$4x + 2y = 8$

is a redundant system $\therefore$ has infinitely many solutions

$\Rightarrow y = -2x + 4$

we might describe its infinite solutions as

$(x, -2x + 4)$

ex In the recent brexit deal vote there were 230 more votes against than there were votes for. There were 634 votes cast. How many voted for, how many against?

- assign variables: let F be # voted for
  A be # voted against

- interpret data:
  - 230 more voted against than for : $A - F = 230$
  - 634 votes were cast : $A + F = 634$

- rearrange & solve!

$$\begin{aligned} A - F &= 230 \\ A + F &= 634 \end{aligned} \xrightarrow{eq2 + eq1} \begin{aligned} A - F &= 230 \\ (A+F) + (A-F) &= 230 + 634 \end{aligned} \Rightarrow \begin{aligned} A - F &= 230 \\ 2A &= 864 \end{aligned}$$

$$\rightarrow \begin{aligned} A - F &= 230 \\ A &= 432 \end{aligned} \xrightarrow{\text{subst } A} \begin{aligned} (432) - F &= 230 \\ A &= 432 \end{aligned} \Rightarrow \begin{aligned} F &= 202 \\ A &= 432 \end{aligned}$$

- State our answer!

  432 voted against, 202 voted for

ex

One batch of cookies requires 3 cups of flour and 1 cup of sugar. One batch of brownies requires 1 cup of flour and 2 cups of sugar. You have 25 cups of flour and 20 cups of sugar. You want to use up all of your sugar and flours. How many batches of cookies and brownies should you make?

- assign variables: let C be # of batches cookies
  B " " brownies

- interpret our data:
  - we have 25 cups of flour, 20 cups of sugar, $\begin{aligned} 3C + 1\cdot B &= 25 \\ 1C + 2B &= 20 \end{aligned}$

- solve:

$$\begin{aligned} 3C + 1B &= 25 \\ 1C + 2B &= 20 \end{aligned} \xrightarrow{-3 * eq2} \begin{aligned} 3C + 1B &= 25 \\ -3C - 6B &= -60 \end{aligned}$$

$$\xrightarrow{eq1 + eq2} \begin{aligned} 3C + 1B &= 25 \\ \cancel{0C} - 5B &= -35 \end{aligned} \Rightarrow \begin{aligned} 3C + 1B &= 25 \\ B &= 7 \end{aligned}$$

$$\xrightarrow[B=7]{\text{plug in}} \begin{aligned} 3C + 1\cdot 7 &= 25 \\ B &= 7 \end{aligned} \Rightarrow \begin{aligned} 3C &= 18 \\ B &= 7 \end{aligned} \Rightarrow \begin{aligned} C &= 6 \\ B &= 7 \end{aligned}$$

- answer question: 6 batches of cookies and 7 batches of brownies uses up all sugar and flour.

Intro to Matrices

A **matrix** is a rectangular array (in this class) of numbers or unknowns

$$M = \begin{bmatrix} 1 & 2 \\ 3 & 7 \\ 1 & 3 \end{bmatrix}$$

is an example matrix.

It has 3 rows, 2 columns, and can be described as a 3 by 2 matrix (3×2 matrix)