Review Problem

Two bank accounts have the following balance formula:

$$B_1(t) = 1500(1.03)^{4t} \qquad B_2(t) = 2100(1.05)^{2t}$$

Which bank account started with the higher balance? Which bank account offers the higher annual interest rate?

Recall interest formula: $A(t) = P\left(1 + \frac{r}{m}\right)^{mt}$

- P is our initial amount
- r is our annual interest rate
- m is # times interest is compounded per year

Bank 1:

$P = 1500$

$m = 4$

$\left(1 + \frac{r}{m}\right) = 1.03$

$1 + \frac{r}{4} = 1.03$

$\frac{r}{4} = 0.03$

$r = 0.12$

Bank 2:

$P = 2100$

$m = 2$

$\left(1 + \frac{r}{m}\right) = 1.05$

$1 + \frac{r}{2} = 1.05$

$\frac{r}{2} = 0.05$

$r = 0.1$

Bank 2 Started w/ higher balance

Bank 1 offers higher annual interest rate

1.3 Linear Functions

Consider the following table

x	0	1	2	3	4	5
$f(x)$	-17	-14	-11	-8	-5	-2

$+3 \quad +3 \quad +3 \quad +3 \quad +3$

What would you guess is $f(5)$?

-2 might be a good guess ... why?

Consider

x	-2	0	1	2	5	6
$g(x)$	9	7	6	5	2	$a=1$

$-2 \quad -1 \quad -1 \quad -3$

at first glance we might not see that same property

<u>however</u> our x's are spaced out different here

Let's look at how $g(x)$ changes <u>as</u> x changes

$$\frac{\text{change in } g}{\text{change in } x}$$

$$\frac{g \text{ value in table} - \text{previous } g \text{ value in table}}{x \text{ value in table} - \text{previous } x \text{ value}}$$

$$\frac{7-9}{0-(-2)} = \frac{-2}{2} = -1$$

$$\frac{6-7}{1-0} = \frac{-1}{1} = -1$$

$$\frac{5-6}{2-1} = \frac{-1}{+1} = -1$$

$$\frac{2-5}{5-2} = \frac{-3}{3} = -1$$

So maybe

$$\frac{a-2}{6-5} = -1$$

$$\frac{a-2}{1} = -1$$

$$a-2 = -1$$

$$a = 1$$

Functions with this constant rate of change are called linear functions.

A linear function is one that can be written as

$$y = mx + b$$

ex

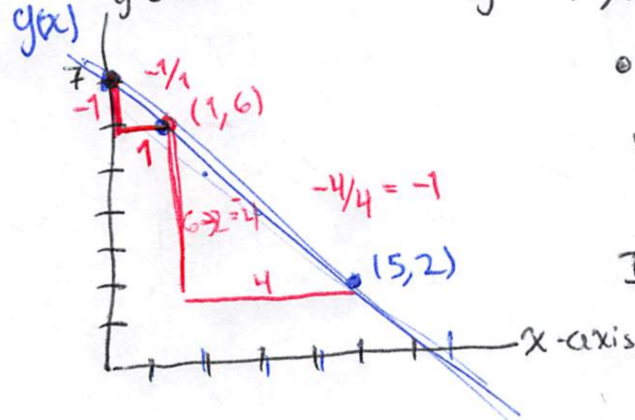
Let's find m and b for $g(x)$.

- $g(0) = 7$ so $7 = g(0) = m * 0 + b = b$
  so $b = 7$

- $g(1) = 6$ so $6 = g(1) = m * 1 + b = m + 7$
  $6 = m + 7$
  $m = -1$

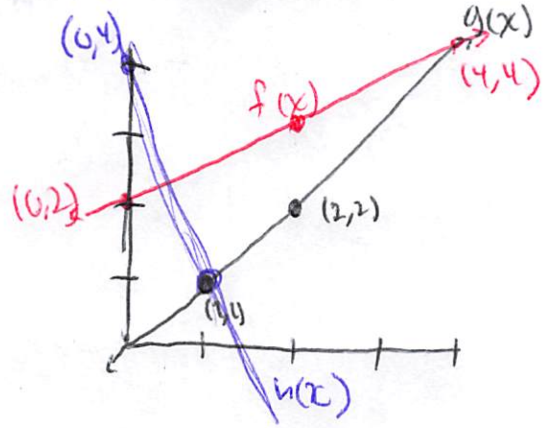
Let's graph of g

$g = -1x + 7$



- Because $g(x)$ 'crosses' the y-axis at 7 we say the y-intercept of g is 7.
  In general b is the y-intercept

- The slope of a line measures 'rise over run', 'change in y / change in x', '$\Delta y / \Delta x$' and in general m is the slope.



- The slope of f is $+1/2$
- The slope of g is $+1/1$
- The slope of h is $-3/1 = -3$

So a 'bigger' slope means 'steeper' line
a negative slope gives a line 'going down'

We know 2 pts define a line and linear functions give lines, so how do we find a linear function from 2 pts?

ex: What is the slope of the line through (1,2) and (6,7)?

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{7-2}{6-1} = \frac{5}{5} = 1$$

What would its ~~ ~~ y-intercept be?

- y-intercept is b in $y = mx + b$
- m was slope; $m = 1$
- $y = 1x + b$
- choose a point on the line, (1,2)
- so $2 = 1 * 1 + b = 1 + b$
  $b = 1$
- The y-intercept is $y = 1$.
- The formula of the line $y = x + 1$

$(1) + 1 = 2$ ✓
$(6) + 1 = 7$ ✓

non-example:

x	0	2	4	6
y	0	0.5	1.5	4.5

If we tried what we've been doing...

$\frac{\Delta y}{\Delta x} = \frac{0.5 - 0}{2 - 0} = \frac{0.5}{2} = 4$, $\frac{\Delta y}{\Delta x} = \frac{1.5 - 0.5}{4 - 2} = \frac{1}{2} = 1/2$

$\frac{\Delta y}{\Delta x} = \frac{4.5 - 1.5}{6 - 4} = \frac{3}{2} = 3/2$ and $4 \neq \frac{1}{2} \neq 3/2$

because the rate of change between multiple points is not same throughout, it is not a linear function.

There does not exist m or b s.t.

(in this instance) $y = mx + b$

Think back to linear cost function

$C(x) = (\text{marginal cost})\, x + \text{fixed cost}$

- we described marginal cost as the increased per item increased $\Rightarrow$ is a rate that describes how $C(x)$ changes as x changes

- fixed cost: the cost regardless of # of items or the cost even if no items were dealt with.

y-intercepts are our fixed costs

Suppose a print shop's costs followed a LCF. If it cost \$1.30 for 10 prints and \$1.72 for 16 prints, how much would 25 prints cost?

- we know our formula looks like $y = mx + b$
- $(10, 1.30)$ 10 prints cost \$1.30
- $(16, 1.72)$

- Let's find m
- $m = \dfrac{\text{change in cost}}{\text{change in \# prints}} = \dfrac{1.72 - 1.30}{16 - 10} = \dfrac{0.42}{6} = 0.07$

- Let's find b
- $y(10) = 1.30 \Rightarrow$

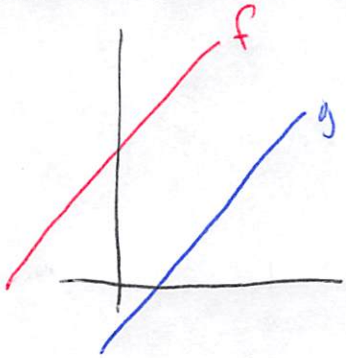
$$1.30 = (0.07) * (10) + b$$
$$1.30 = 0.70 + b$$
$$b = 0.60$$

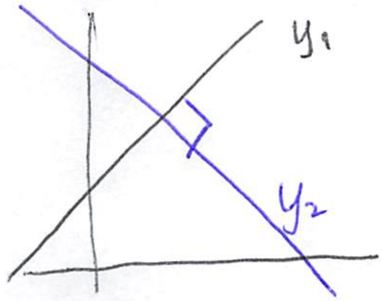
- $y = 0.07x + 0.60$

$y(25) = 0.07 * (25) + 0.60 = 1.75 + 0.60 = 2.35$

25 prints cost \$2.35; each print was 7¢ but we pay \$0.60 to access printer

Interpreting parallel and perpendicular





we say 2 lines are parallel if they would (when extended) never intersect and perpendicular if they intersect at 90° angles.

f and g are parallel

y_1 and y_2 are perpendicular

parallel: parallel lines have equal slope; equal steepness or equal rate of change

perpendicular: if the slope of y_1 is m then the slope of y_2 is $-1/m$,

we call this the negative reciprocal

ex:

Find a line parallel to $y = 10x + 2$ that goes through $(3, 7)$.

- parallel lines have equal slope
- in $y = mx + b$, m is slope
- our slope is therefore going to be 10
- In general lines look like $y = mx + b$
- Our line goes through $(3, 7)$

• So $7 = m * 3 + b$

so $7 = 10 * 3 + b$

$7 = 30 + b$

$b = -23$

• So the line going through (3, 7), parallel to $y = 10x + 2$ has the formula

$$y = 10x - 23.$$

ex: Find when the line through (-1, 2) and (3, -1) intersects the line through (2, 1) and (6, 4).

• Find equations for the 2 lines

• $y_1 = m_1 x + b_1$

(-1, 2) and (3, -1)

$$m_1 = \frac{rise}{run} = \frac{-1 - 2}{3 - (-1)} = \cancel{-3} \; \frac{-3}{4} = -3/4$$

using (-1, 2) $\quad 2 = (-3) * (-1) + b_1$

$2 = 3 + b$

$b_1 = -1$

• $y_2 = m_2 x + b_2$

$m_2 = \frac{4 - 1}{6 - 2} = 3/4$; using (2, 1) $\quad 1 = (\frac{3}{4}) * (2) + b_2$

$1 = \frac{3}{2} + b \Rightarrow b = -1/2$

we want to find $y_1 = y_2$

find $(-\frac{3}{4}) x - 1 = \frac{3}{4} x - 1/2 \ldots$