Warm-up / Review:

Let $f(x) = x^2 + x$ and $g(x) = 2x + 2$

Find:

$f(1) = (1)^2 + (1) = 2$

$g(1) = 2(1) + 2 = 4$

$f(g(1)) = f(4) = (4)^2 + 4 = 20$

$f(x) - g(x) = (x^2 + x) - (2x + 2) = x^2 - x - 2$

$f(a+h) = (a+h)^2 + (a+h) = a^2 + 2ah + h^2 + a + h$

$g(f(x)) = 2(x^2 + x) + 2$

$= 2x^2 + 2x + 2$

1.2 Lots of Models

Different models for different Situations

ex The cost of a bird scooter is \$1 to unlock and \$0.15 per minute.

How much does 1 minute ride cost? $1 + 1\cdot(0.15)$

2 $1 + 2(0.15)$

3 $1 + 3(0.15)$

That cost as a function of minutes t is

$$C(t) = 1 + 0.15t$$

Suppose average electric scooter costs \$400 and average bird ride was for 10 minutes. How many average rides to make up cost?

$$\$400 = \left(\text{How much made from average ride}\right) r$$

Average ride costs $1 + 0.15(10) = 2.50$

$400 = 2.50r$ gives us $r = 160$

To say 160 rides to reach cost of scooter

Cost, Revenue, Profit

<u>Cost function</u>, $C(x)$, specifies the cost as a function on some # of units or items x. Often of form

$$\text{Cost} = \underset{\text{is not ~~deter~~ changed by } x}{\text{fixed cost}} + \underset{\text{changes / varies as } x \text{ increases or decreases}}{\text{variable cost}}$$

A $C(x)$ of the form

$$C(x) = b + mx$$

we call it a <u>linear cost function</u> and m is called <u>marginal cost</u>, a measure of the incremental cost per item.

in bird ex. we had

$$C(x) = \overbrace{1}^{\text{fixed cost}} + \overbrace{\underbrace{0.15}_{\text{marginal cost}} +}^{\text{variable cost}}$$

<u>Revenue</u> or net sales measures how much money is brought in. If an item sells at cost n then $R(x) = nx$ (often)

<u>Profit</u> or net <u>income</u>, total amount made

$$\text{Profit} = \text{Revenue} - \text{Cost}$$

$P(x) = R(x) - C(x)$

A negative profit relates to a net loss

When $R(x) = C(x)$ we have a break-even point

Note $R(x) = C(x)$ then $P(x) = 0$.

ex An online entrepeneur paid a web designer \$630 to build a site and monthly hosting costs \$30. Through website an average \$120 per month is made.

when (~~does~~?) does this entrepeneur break even?

$P(x) = 0$ or $R(x) = C(x)$ then we need to find R, C

Cost $= 630 + 30m$

Rev. $= 120m$

$$\text{Profit} = \underset{R}{(120m)} - \underset{C}{(630 + 30m)} = 90m - 630$$

When does Profit $= 90m - 630 = 0$?

$$90m - 630 = 0$$
$$90m = 630$$
$$m = 7$$

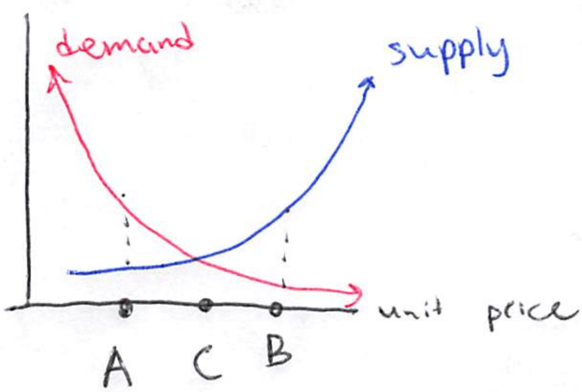
- Is the long term trend profitable?

yes; after 7 months they will be making a profit

Supply and Demand

A demand function expresses the number of items demanded, q, as a function of unit price.

A supply function expresses the number of items a supplier can make available, q, as a function of unit price p.



- at p = A there is more demand than can be supplied ie a Shortage
- at p = B, greater supply than demand, ie a surplus
- at p = C we have an equilibrium. C is the equilibrium price and q(C) is equilib demand

ex If the demand for calculators is $-5p + 800$ and the supply is $15p - 400$, then what is the optimal selling price?

OSP happens at equilibrium

equilibrium happens when supply equals demand

So we are looking for p such that

$$-5p + 800 = 15p - 400$$

$$1200 = 20p$$

$$p = 60$$

Calculators should be sold at \$60

Modeling Interest

Suppose you invest \$1000 into an account with annual interest of ~~5%~~ 15% compounded annually. How does it grow with time.

t # years	0	1	2	3
balance	1000	$1000(1.15)$ $= 1150$	$1150(1.15) =$ $1000(1.15)(1.15)$ $1000(1.15)^2$ $= 1322.5$	$1322.5(1.15) =$ $1000(1.15)^3$

Let $A(t)$ be balance

Then
$$A(1) = 1000(1.15)^1$$
$$A(2) = 1000(1.15)^2$$
$$A(3) = 1000(1.15)^3$$

and in fact
$$A(t) = 1000(1.15)^t$$

In general:

- if we start with an initial amount $\underline{P}$
- investing for $\underline{t}$ many years
- have an interest rate of $\underline{r}$ (annual rate)
- have interest compounded $\underline{m}$ times a year

then
$$A(t) = P\left(1 + \frac{r}{m}\right)^{mt}$$

ex in our past ex what was P, r, and m?

P was 1000, $r = 0.15$, $m = 1$

was $A(t) = 1000\left(1 + \frac{0.15}{1}\right)^{1 \cdot t}$? yes ✓

ex Cynthia has \$1000 and can open an account in 1 of 2 banks.

Bank 1: has an initial fee of \$200 and has 7% interest compounded monthly

Bank 2: no opening cost, 6% annual interest, compounded twice a year.

If this first deposit (and only) what would the balances be after 10 yrs? 20 yrs?

Bank 1: $P = 800$, $r = 0.07$, $m = 12$

Bank 2: $P = 1000$, $r = 0.06$, $m = 2$

$$A_1(t) = 800\left(1 + \frac{0.07}{12}\right)^{12t} \qquad A_2(t) = 1000\left(1 + \frac{0.06}{2}\right)^{2t}$$

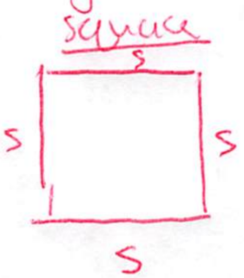
$A_2 > A_1$ for $t < \sim 21$ and $A_1 > A_2$ for $t > \sim 21$

Models using common formulas:

perimeter, area, $a^2 + b^2 = c^2$

ex You have 80 ft of fencing and want to make a square enclosure. What is the area of the largest enclosure we fence in?

☆ diagram whenever possible ☆



fencing → perimeter

$P = 4s$

area

$A = s^2$

max perimeter is 80 ft

so $80 = 4s$

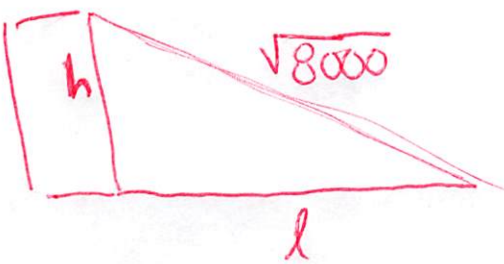
so $s = 20$

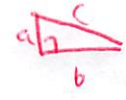
so $A = (20)^2 = 400$

So $400\ ft^2$ is largest enclosed square.

ex. At 6pm a building's shadow is twice as long as the building's height. The distance from top of the building to the end of the shadow $\sqrt{8000}$ ft ≈ 90 ft.

How tall is the building?



remember that in 

$a^2+b^2=c^2$

so $h^2+l^2=(\sqrt{8000})^2$

$h^2+l^2=8000$

* $l=2h$

so $(h)^2+(2h)^2=8000$

$h^2+4h^2=8000$

$5h^2=8000$

$h^2=1600$

$h=40$

The building is 40 ft tall.

A label company costs a fee of $10 a month and 5% of total revenue. Your band receives $.006 a stream. How many streams are needed to be profitable?
per month

$C = 10 + 0.05R$

$R = 0.006(s)$ $\Rightarrow C = 10 + 0.05(0.006s)$

$P = R - C$

$P = 0.006(s) - (10 + 0.0003s)$

¿ when is $P > 0$?