Functions

Consider the following table

year t (since 2004)	0	1	2	3	4	5	6
# of facebook users (by millions)	0	1	5.5	12	58	150	450

so 1 year after 2004, approx 1 million users
3 years after 2004, approx 12 million users
4 " " 58 " "

we can describe this relationship as a function

define $n(t)$ 'n of t' as # of million fb users t years after 2004
so...

$n(0) = 0$ $\quad n(1) = 1$ $\quad n(2) = 5.5$
$n(4) = 58$ $\quad n(5) = 150$ $\quad n(6) = 450$

A <u>function</u> takes inputs and returns outputs
* such that any input has only one output *

<u>ex</u>: barcodes → a single barcode only ever scans as one thing
url → only ever 1 website

we will focus mostly on functions whose inputs and outputs are real #'s (think $0, 1, 2, 5/2, -4, 0.1327\ldots, \sqrt{17}, \pi$) we'll call these real-valued functions of a real variable

we call the set of inputs of a function its <u>domain</u>

ex. $n(2)$ was defined as 5.5 2 was in the dom
$n(-2)$ was not defined and therefore not in te dom.

Let $s(m)$ be the function that tells how many students born on mth day of the month.

here the domain of s is $1, 2, 3, \ldots, 31$

but 2.5, 13.3, or 33 would not be in domain.

we can also define functions with an equation.

ex $f(x) = 2x + 1$

$f(1) = 2(1) + 1 = 3$

$f(2) = 2(2) + 1 = 5$

$f(\pi) = 2(\pi) + 1 = 2\pi + 1$

$f(a) = 2(a) + 1 = 2a + 1$

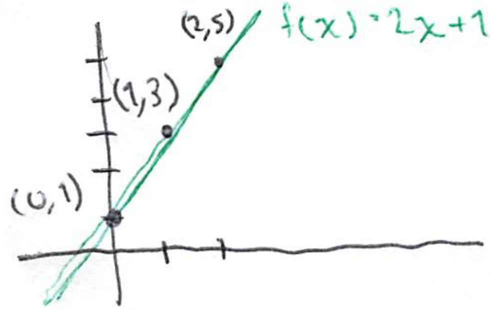
$f(3.5) = 2(3.5) + 1 = 8$

$f(-4) = 2(-4) + 1 = -7$

$f(0) = 2(0) + 1 = 1$

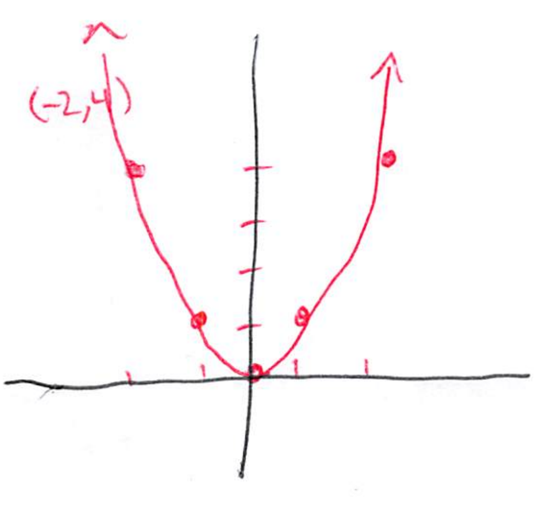
* domain of f is all real numbers *

we can associate these input and outputs (x and $f(x)$) as ordered pairs $(x, f(x))$ and treat them as points. we can graph points



Let's try $g(x) = x^2$

x	$g(x)$	$(x, g(x))$
-2	4	(-2,4)
-1	1	(-1,1)
0	0	(0,0)
1	1	(1,1)
2	4	(2,4)



So far we've been describing functions as '$f(x) = \dots$' where x is the <u>independent variable</u>

but sometimes we omit the '(x)'

ex. $y = 5x + 4$

- technically just an equation
- we can treat this as a function

we don't need to use x as ind. variable

<u>ex</u> Suppose that notebooks cost \$1 each for the first 10, but afterwards are \$0.75 each

Let $C(n)$ be the cost of n notebooks.

$C(n) = 1 \cdot n$ is kinda correct → only good for $n \leq 10$

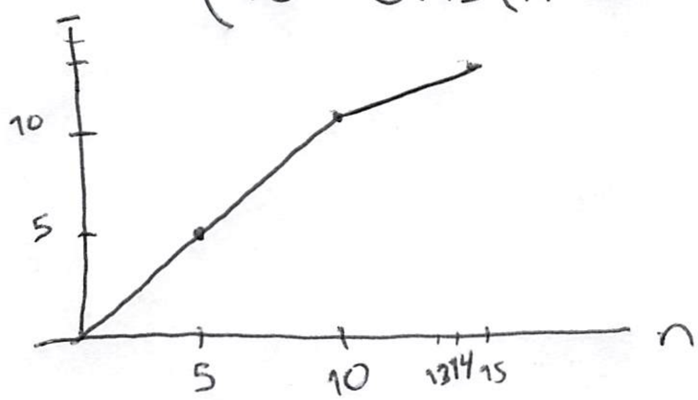
1 notebook costs \$1
2 \$2
⋮
5 \$5

11 should be $10.75 = 10 \cdot 1 + .75$
12 $11.50 = 10 \cdot 1 + 2(.75)$
14 $13 = 10 \cdot 1 + 4(.75)$

$C(n) = 10 + 0.75(n-10)$ → only good for $n \geq 10$

We can use a piecewise function

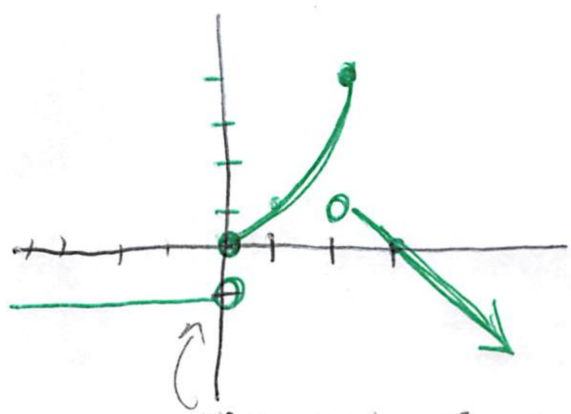
$$c(n) = \begin{cases} 1 \cdot n & n \leq 10 \\ 10 + 0.75(n-10) & n > 10 \end{cases}$$



A piecewise function contains conditional rules.
A given input will satisfy a condition and the output will be the eval. of assoc. rule

$$g(x) = \begin{cases} -1 & x < 0 \\ x^2 & 0 \leq x \leq 2 \\ -x+3 & 2 < x \end{cases}$$

$g(-1) = -1$
$g(0) = 0 = (0)^2$
$g(1) = 1 = (1)^2$
$g(2) = 4 = (2)^2$
$g(3) = -(3)+3 = 0$



open circle if $<, >$
closed circle if $\leq, \geq$

for Desmos.com

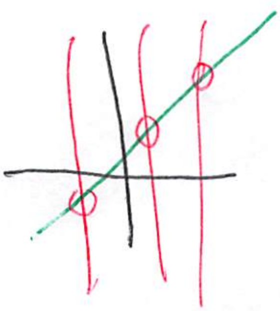
$y = \{ \cancel{x}\text{condition1} : \text{rule1}, \text{cond2} : \text{rule2} \}$

Recall: a function has only one output for every one input

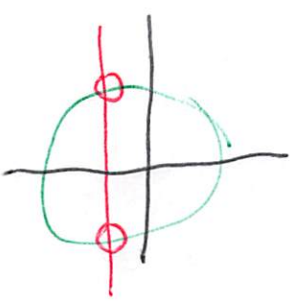
If using graphs to describe functions then this notion can be described as the <u>vertical line test</u>

VLT: if a vertical line cross a graph in more than 1 location, then the graph is not of a function

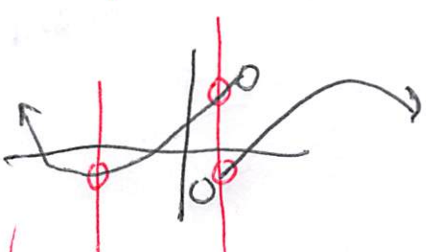
ex



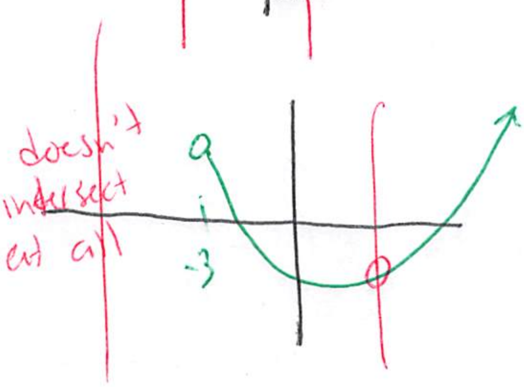
every vert. line only intersects once ✓
passes VLT; it is a graph of a function



there exists a vertical line that crosses twice
therefore VLT it is <u>not</u> a graph of a function



✗ does not pass VLT is not graph of a function.



✓ still satisfies VLT; is a function